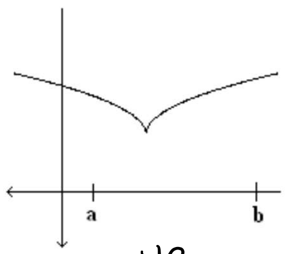
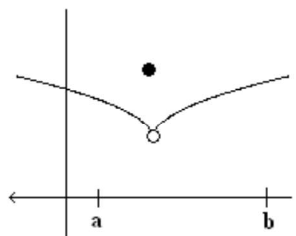
# Unit 5.5 Candidates' Test

1. For which of the following functions is the Extreme Value Theorem NOT APPLICABLE on the interval [*a*, *b*]? Give a reason for your answer.

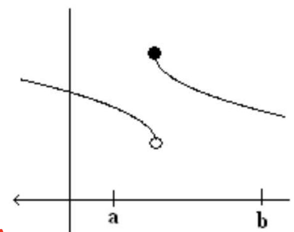
Graph I




Graph II




Graph III



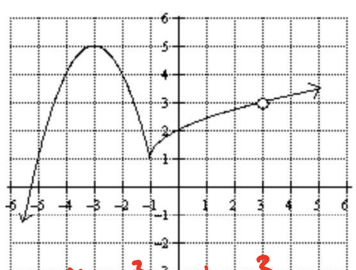

Applicable

Graph II and III are not continuous on [a, b]

∴ The E.V.T does not apply.

For exercises 2 – 4, determine the critical numbers for each of the functions below.

2.



$x = -3, -1, 3$

$f'(x) = 0$ at $x = -3$

$f'(x) =$ und at $x = -1, 3$

3. $g(x) = \ln(x^2 + 4)$

$g' = \frac{2x}{x^2+4}$

$g' = 0$: $2x = 0$, $x = 0$

$g' =$ und: $x^2 + 4 = 0$, $x = \pm 2i$ (not real)

$\therefore x = 0$

4. $h(x) = \sqrt[3]{x+3}$ $= (x+3)^{1/3}$

Chain

$h' = \frac{1}{3}(x+3)^{-2/3}(1)$

$h' = \frac{1}{3(x+3)^{2/3}}$

$h' = 0$: $0 = 1$ No solution

$h' =$ und: $3(x+3)^{2/3} = 0$, $x = -3$

$\therefore x = -3$

5. Given the function below, apply the Extreme Value Theorem to find the absolute extrema of *f*(*x*) on the indicated interval.

Point

CALC

$f(x) = \sin x \cdot \ln(x+1)$ on the interval [1, 6]

$f' = \cos x \cdot \ln(x+1) + \sin x \cdot \frac{1}{x+1}$

$f' = \cos x \cdot \ln(x+1) + \frac{\sin x}{x+1}$

$f' = 0$ (calc): $x = 1.887$, $x = 4.810$

$f' =$ und: $x + 1 = 0$, $x = -1$; $x + 1 \le 0$, $x \le -1$ — not in Domain

CALC

| $x$ | $f(x)$ |
|---|---|
| 1 | 0.583 |
| 1.887 | 1.008 |
| 4.810 | -1.751 |
| 6 | -0.544 |

∴ ABSOLUTE MAX @ (1.887, 1.008)

ABSOLUTE MIN @ (4.810, -1.751)

For exercises 6 – 9, determine the absolute extreme values on the given interval. You should do each of these independent from a calculator.

6. $f(x) = x^3 - 3x^2$ on the interval [–1, 3]

$f' = 3x^2 - 6x$

| $f' = 0$ | $f' =$ und |
|---|---|
| $3x(x-2)=0$ | $\emptyset$ |

$x = 0, x = 2$

CANIDATES

| x | f(x) |
|---|---|
| -1 | 4 |
| 0 | 0 |
| 2 | -4 |
| 3 | 0 |

$f(-1) = (-1)^3 - 3(-1)^2 = -1 - 3 = -4$

$f(0) = (0)^3 - 3(0)^2 = 0$ Reference

$f(2) = (2)^3 - 3(2)^2 = 8 - 3(4) = 8 - 12 = -4$

$f(3)\ (3)^3 - 3(3)^2 = 27 - 27 = 0$

∴ ABSOLUTE MAX: Y=0
ABSOLUTE MIN: Y=-4

7. $g(x) = \sqrt[3]{x+2} = (x+2)^{1/3}$ on the interval [–3, 6]

$g' = \frac{1}{3}(x+2)^{-2/3}(1)$

$g' = \dfrac{1}{3(x+2)^{2/3}}$

| $g' = 0$ | $g' =$ und |
|---|---|
| no solution | $3(x+2)^{2/3} = 0$ |
| | $x = -2$ |

CANIDATES

| x | g |
|---|---|
| -3 | -1 |
| -2 | 0 |
| 6 | 2 |

Reference

$g(-3) = \sqrt[3]{(-3)+2} = \sqrt[3]{-1} = -1$

$g(-2) = \sqrt[3]{(-2)+2} = \sqrt[3]{0} = 0$

$g(6) = \sqrt[3]{(6)+2} = \sqrt[3]{8} = 2$

∴ ABSOLUTE MAX: Y=2
ABSOLUTE MIN: Y=-1

8. $h(x) = \frac{x}{x+2}$ on the interval [–4, 0]

h(x) is not continuous on [-4,0] due to VA @ x=-2. ∴ The E.V.T. does not apply for h(x) on [-4,0]

9. $f(x) = 3x^{\frac{2}{3}} - 2x$ on the interval [–1, 1]

$f' = 2x^{-1/3} - 2$

$f' = \dfrac{2}{x^{1/3}} - 2 \cdot \dfrac{x^{1/3}}{x^{1/3}}$

$f' = \dfrac{2 - 2x^{1/3}}{x^{1/3}}$

| $f' = 0$ | $f' =$ und |
|---|---|
| $2 - 2x^{1/3} = 0$ | $x^{1/3} = 0$ |
| $1 = x$ | $x = 0$ |

CANIDATES

| x | f(x) |
|---|---|
| -1 | 5 |
| 0 | 0 |
| 1 | 1 |

$f(-1) = 3\sqrt[3]{(-1)^2} - 2(-1) = 3 + 2 = 5$

$f(0) = 3\sqrt[3]{0^2} - 2(0) = 0 - 0 = 0$

$f(1) = 3\sqrt[3]{1^2} - 2(1) = 3 - 2 = 1$

∴ ABSOLUTE MAX: Y=5
ABSOLUTE MIN: Y=0