Unit 5.6 Determining Concavity of Function over Its Domain

APClassroom Hw 5.6

1. The second derivative of the function g is given by $g''(x) = x^5 - 2.2x^4 - 6.61x^3 + 8.602x^2$. At which values of x in the interval $-3 < x < 4$ does the graph of g have a point of inflection where the concavity of the graph changes from concave up to concave down?

(A) $x = 1.1$ only

(B) $x = -2.3$ and $x = 3.4$ only

(C) $x = -2.3$, $x = 1.1$, and $x = 3.4$ only

(D) $x = -2.3$, $x = 0$, $x = 1.1$, and $x = 3.4$

g'' is + then g'' is −

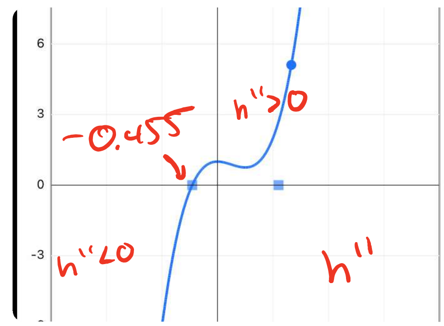
2. The first derivative of the function h is given by $h'(x) = x^4 - x^3 + x$. On which of the following intervals is the graph of h concave down?

(A) $(-0.755, 0)$

(B) $(0, 0.5)$ only

(C) $(-0.455, \infty)$

(D) $(-\infty, -0.455)$



3. At what values of x does the graph of $y = x^2 e^{-2x}$ have a point of inflection?

$y''=0$, y'' undefined

(A) $x = -2$ and $x = 0$

(B) $x = 0$ and $x = 1$

(C) $x = -2 - \sqrt{2}$ and $x = -2 + \sqrt{2}$

(D) $x = 1 - \frac{\sqrt{2}}{2}$ and $x = 1 + \frac{\sqrt{2}}{2}$

$y' = 2x\,e^{-2x} + x^2 e^{-2x}(-2)$

$y'' = 2 \cdot e^{-2x} + 2x e^{-2x}(-2) + 2x \cdot e^{-2x}(-2) + x^2 e^{-2x}(-2)(-2)$

$y'' = e^{-2x}\left[2 + 2x(-2) + 2x(-2) + x^2(-2)(-2)\right]$

$y'' = e^{-2x}\left[4x^2 - 8x + 2\right]$

$y'' = 2e^{-2x}(2x^2 - 4x + 1)$

$(2e^{-2x}$ never $= 0)$ $\quad 2x^2 - 4x + 1 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

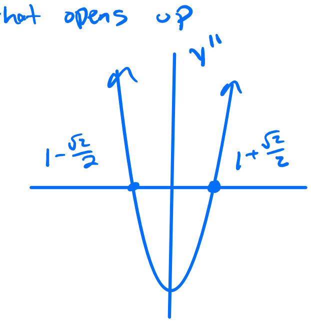
$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}$

$x = \dfrac{4 \pm \sqrt{8}}{4}$

$x = \dfrac{4 \pm 2\sqrt{2}}{4}$

$x = 1 \pm \frac{\sqrt{2}}{2}$

ProTip $2x^2 - 4x + 1$ is a parabola that opens up



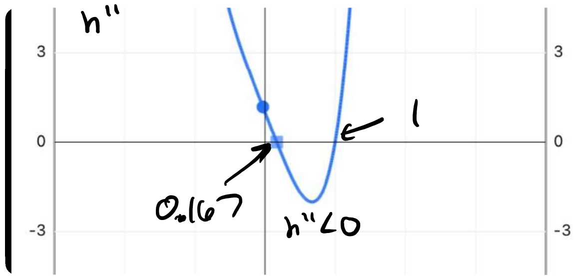
4. The first derivative of the function h is given by $h'(x) = x^5 - 3x^2 + x$. What are all intervals on which the graph of h is concave down?

(A) $(-\infty, 0)$ and $(0.338, 1.307)$

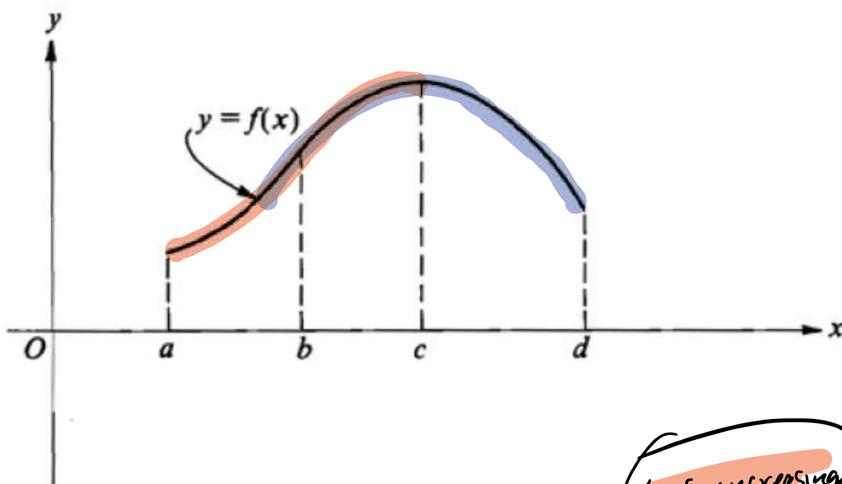
(B) $(-\infty, 0.669)$

(C) $(-\infty, 0.167)$ and $(1, \infty)$

(D) $(0.167, 1)$



5.



The graph of $y = f(x)$ is shown in the figure above. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?

Y is increasing

Y is concave down

I.a<x<b

II.b<x<c

III.c<x<d

(A) I only

(B) II only

(C) III only

(D) I and II

(E) II and III

6.

x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

2.0 1.8

Let f be a function such that $f''(x) < 0$ for all x in the closed interval [1, 2]. Selected values of f are shown in the table above. Which of the following must be true about $f'(1.2)$?

(A) $f'(1.2) < 0$

(B) $0 < f'(1.2) < 1.6$

(C) $1.6 < f'(1.2) < 1.8$

(D) $1.8 < f'(1.2) < 2.0$

(E) $f'(1.2) > 2.0$

7.

Graph of f'

The graph of f', the derivative of the function f, is shown in the figure above. Which of the following statements about f at $x = -2$ is true?

(A) f is not continuous at $x = -2$.

(B) f has an absolute maximum at $x = -2$.

(C) The derivative of f does not exist at $x = -2$.

(D) The graph of f has a point of inflection at $x = -2$.

(E) The graph of f has a vertical tangent line at $x = -2$.

8.

x	0	1	2	3
$f''(x)$	5	0	−7	4

We don't know where $f'(x)=0$!

The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

we don't know what is happening between the x-values.

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

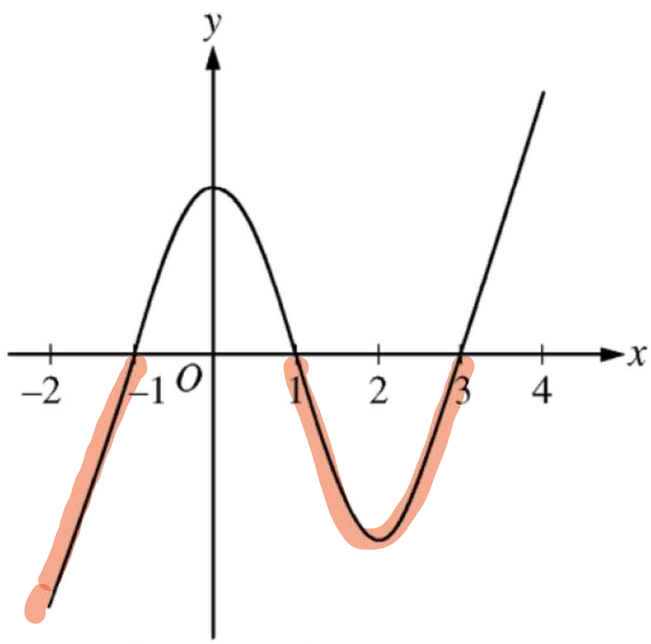
(C) f has a local maximum at $x = 1$

(D) The graph of f has a point of inflection at $x = 1$

(E) The graph of f changes concavity in the interval $(0, 2)$.

b/c f'' changes sign between 1 and 2

9.



Graph of f''

The graph of f'', the second derivative of f, is shown above for $-2 \le x \le 4$. What are all intervals on which the graph of the function f is concave down? when $f'' < 0$

(A) $-1 < x < 1$

(B) $0 < x < 2$

(C) $1 < x < 3$ only

(D) $-2 < x < -1$ only

(E) $-2 < x < -1$ and $1 < x < 3$

10. An equation of the line tangent to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$

1) FIND PoI

$y' = 3x^2 + 6x$

$y'' = 6x + 6$

CV: $x = -1$

(B) $y = -3x + 1$

$y'' = 6(x+1)$

(C) $y = 2x + 10$

NEG -1 POS

(D) $y = 3x - 1$

PoI @ $x = -1$

2) Tangent line

(E) $y = 4x + 1$

PoT $(-1,4)$	SoT	tangent
$y(-1) = (-1)^3 + 3(-1)^2 + 2$ $y(-1) = 4$	$y'(-1) = 3(-1)^2 + 6(-1)$ $= -3$	$y - 4 = -3(x+1)$ $y = -3x - 3 + 4$ $y = -3x + 1$

11. The second derivative of a function f is given by $f''(x) = \sin(3x) - \cos(x^2)$. How number of points of inflection does the graph of f have on the interval $0 < x < 3$?

(A) One

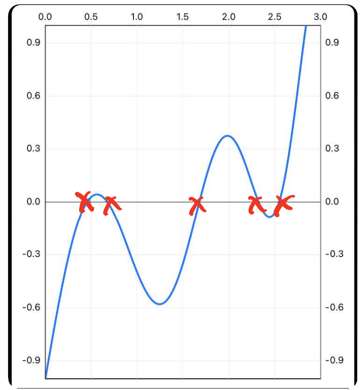
(B) Three

(C) Four

(D) Five

window $[0,3]$

$[1, -1]$



12. The first derivative of the function f is given by $f'(x) = 3x^4 - 12x^3$. What are the x-coordinates of the points of inflection of the graph of f?

(A) $x = 3$ only

(B) $x = 4$ only

(C) $x = 0$ and $x = 2$

(D) $x = 0$ and $x = 3$

(E) $x = 0$ and $x = 4$

$f'' = 12x^3 - 36x^2$

$f'' = 12x^2(x-3)$

$f'' = 12x^2\,(x-3)$

Always +

NEG 0 NEG 3 POS

$x = 3$ Only

$x \neq 0$

13. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0

(B) 1

(C) 2

(D) 3

(E) At no value of x

$y = x^{-2} - x^{-3}$

$y' = -2x^{-3} + 3x^{-4}$

$y'' = 6x^{-4} - 12x^{-5}$

$y'' = \frac{6}{x^4} - \frac{12}{x^5}$

$y'' = \frac{6x-12}{x^5} \Rightarrow x=0,\ x=2$

y'' POS $x=0$ NEG 2 POS

VA on y

Point of inflection on y