Big 10: (Topic 4.2) Particle Motion Name:

Directions: In the box below are the numbers 0–9. Complete the following problems and cross off the number for each answer. If you complete all problems correctly, you will cross off each number exactly once!

0 1 2 3 4 5 6 7 8 9

a) A particle moves along the x axis for $t \geq 0$. The position of the particle is given by $x(t) = t^3 - 9t^2 - 21t + 6$. At what time t does the particle change directions?

① → when velocity changes signs
or
→ when position changes from inc to dec
or
dec to inc

②
$v(t) = 3t^2 - 18t - 21$
$0 = 3(t^2 - 6t - 7)$
$0 = 3(t+1)(t-7)$
$\cancel{t = -1}$ } $t = 7$
not in domain

③ $(t+1)(t-7)$
v: $t=6$ | 7 | $t=8$
$- - - - -$ $+ + + + + +$

④ $v(t)$ changes signs at $\boxed{t = 7}$
∴ particle changes directions there

b) A bug is moving back and forth on a straight path. The velocity of the bug is given by $v(t) = t^2 - 3t$. Find the average acceleration of the bug over the interval $[1, 4]$.

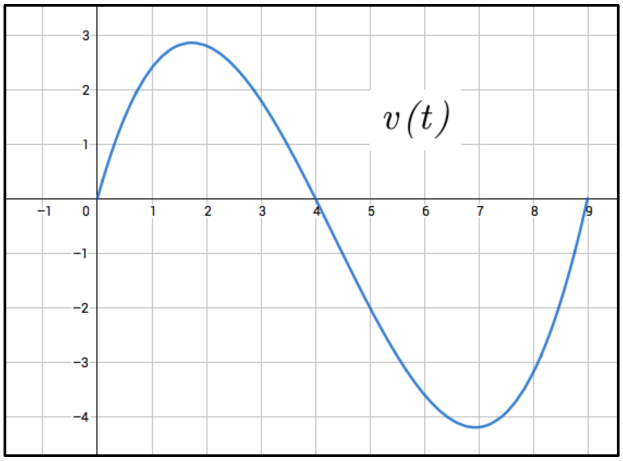
Average Accel (ARC) $= \dfrac{v(1) - v(4)}{1 - 4} = \dfrac{(-2) - 4}{-3} = \dfrac{-6}{-3} = \boxed{2}$

$v(1) = 1^2 - 3(1)$
$v(1) = -2$
$v(4) = 4^2 - 3(4)$
$= 16 - 12$
$v(4) = 4$

$v(t)$ is above x-axis
∴ $v(t) > 0$

$v(t)$ is on x-axis
∴ $v(t) = 0$

$v(t)$ is below x-axis
∴ $v(t) < 0$



$v(t)$ is increasing
∴ $a(t) > 0$

$v(t)$ is decreasing
∴ $a(t) < 0$

$v(t)$ has local extrema
∴ $a(t) = 0$

c) The velocity of a particle for $0 \leq t \leq 9$ is given in the graph above. At which of the following values of t is the particle speeding up?

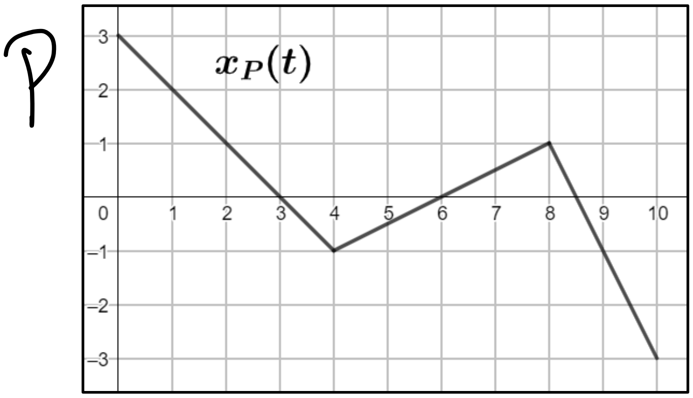
$v(t)$ & $a(t)$ are same sign

$t = 3$
$v(3) > 0$
$a(3) < 0$

$t = 4$
$v(4) = 0$
$a(3) < 0$

$\boxed{t = 5}$
$v(5) < 0$
$a(5) < 0$

$t = 7$
$v(7) < 0$
$a(7) = 0$

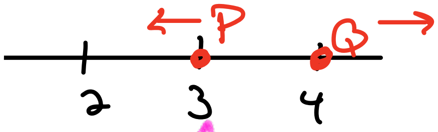


Q

t	1	3	7	9
$x_Q(t)$	4	1	−1	−2
$v_Q(t)$	3	0	−2	3
$a_Q(t)$	−1	−2	4	1

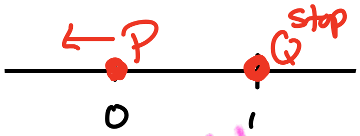
d) For $0 \leq t \leq 10$, particles P and Q move along the x axis. The position of particle P can be modeled by $x_P(t)$ as shown in the figure above. The position of particle Q is defined by $x_Q(t)$. Selected values of $x_Q(t)$, $v_Q(t)$, and $a_Q(t)$ are given in the table above. At what time t are particles P and Q moving towards each other?

$t = 1$



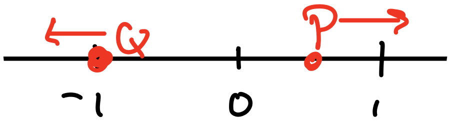
Away

$t = 3$



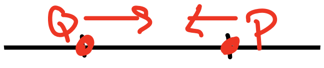
meh

$t = 7$



Away

$t = 9$



Toward

towards @ $t=9$

e) The position of a bug moving along a vertical post is given by the equation $y(t) = \frac{-12}{\pi}\cos\left(\frac{\pi t}{6}\right) + 6t - 1$. Find $v(3)$.

$v = y'(t)$

$$y'(t) = \frac{-12}{\pi} \cdot \underbrace{\left(-\sin\left(\frac{\pi}{6}t\right)\right) \cdot \frac{\pi}{6}}_{\text{chain}} + 6$$

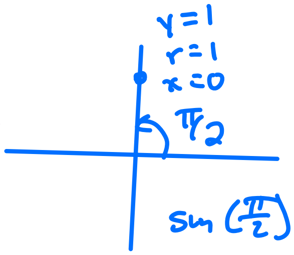
$$v'(t) = 2\sin\left(\frac{\pi}{6}t\right) + 6$$

$$v'(3) = 2\sin\left(\frac{\pi}{6}\cdot 3\right) + 6$$

$$v'(3) = 2\sin\left(\frac{\pi}{2}\right) + 6$$

$$= 2 \cdot 1 + 6$$

$$\boxed{v'(3) = 8}$$



$\sin\left(\frac{\pi}{2}\right) = \frac{y}{r} = 1$

t	1	2	4	6	8
$x(t)$	3	0	-1	-4	5
$v(t)$	2	1	-3	3	0

f) A particle is moving along the x axis. The position and velocity of the particle is recorded for various times in the table above. At which time t is the particle moving towards the origin?

At $t = 6$ ...

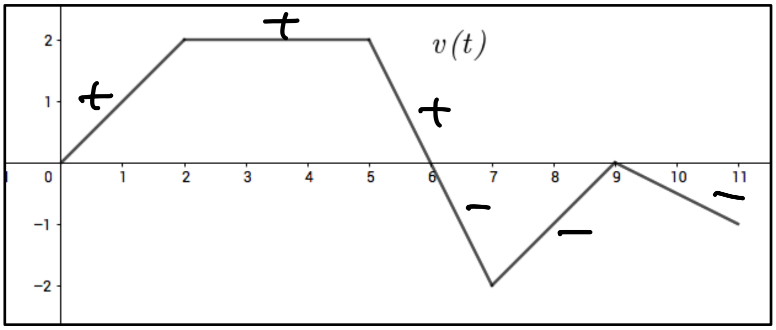
$v(6) = 3$

Position on right and moving left
$\therefore x(t) > 0$ $\therefore v(t) < 0$

OR

Position on left and moving right
$\therefore x(t) < 0$ $\therefore v(t) > 0$

TL:DR $x(t)$ and $v(t)$ are opposite signs



g) The velocity of a particle is given in the graph above for $0 \le t \le 11$. How many times on the interval does the particle change directions?

$v(t) = 0$
$v(t)$ changes signs } particle changes directions

$v(t) = 0$ and changes signs at $t = 6$

$\therefore$ particle changes directions there

The particle changes direction 1 time

h) The position of a bug moving along a straight path is given by $s(t) = t^2 - 2t + 3$. At what time t is the instantaneous velocity equal to the average velocity of the bug on $[0, 6]$?

Mean Value Theorem
(learn later)

$v(t) = 2t - 2$

$\text{A. velocity} = \frac{s(0) - s(6)}{0 - 6}$

$v(t) = \text{Average Velocity}$

$2t - 2 = \frac{s(0) - s(6)}{0 - 6}$

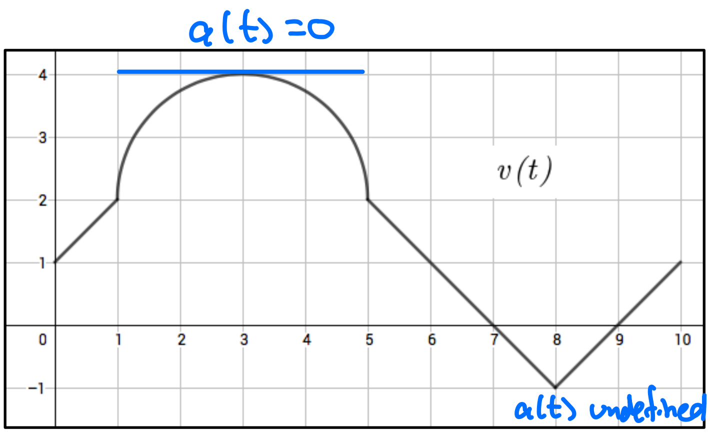
$2t - 2 = \frac{3 - 27}{-6}$

$2t - 2 = \frac{-24}{-6}$

$2t - 2 = 4$

$2t = 6$

$\boxed{t = 3}$



i) The velocity of an object is graphed above for $0 \le t \le 10$. What is the velocity of the object when the acceleration equals 0?

- Acceleration measures rate of change in velocity
- a(t) is slope of velocity
  - a(t) = 0 when horizontal tangent
    - horizontal tangent at SMOOTH Local Max/Min

$t = 3$

$v(3) = \boxed{4}$

j) The position of a bug moving along the x axis is given by $x(t) = at + b$, where a and b are both nonzero. Find the acceleration of the bug at any given time t.

$v(t) = a$ constant

$a(t) = \boxed{0}$