APC - Advanced Techniques
6.1 – Explicit vs Implicit Differentiation

Explicit vs Implicit

Explicit Function: A function written in the form y = *f(x),* where y is defined in terms of x alone.

If $x^2 + y^2 = 100$ find y' using explicit differentiation.

$$y^2 = 100 - x^2$$

$$\frac{d}{dx} y = \frac{d}{dx}(100 - x^2)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(100 - x^2)^{-\frac{1}{2}} \frac{d}{dx}(100 - x^2)$$

$$y' = \frac{1}{2\sqrt{100 - x^2}}(-2x)$$

$$y' = \frac{-2x}{2\sqrt{100 - x^2}}$$

$$y' = \frac{-x}{\sqrt{100 - x^2}}$$

Find the slope of the circle $x^2 + y^2 = 100$ at the point (6, 8)

$$y'(6,8) = \frac{-6}{\sqrt{100 - (6)^2}}$$

$$= \frac{-6}{\sqrt{100 - 36}}$$

$$= \frac{-6}{\sqrt{64}}$$

$$= \frac{-6}{8}$$

$$y'(6,8) = \frac{-3}{4}$$

Find the slope of the circle $x^2 + y^2 = 9$ at the point (-6, 8)

$$y'(-6,8) = \frac{-(-6)}{\sqrt{100 - (-6)^2}}$$

$$= \frac{6}{\sqrt{100 - 36}}$$

$$= \frac{6}{\sqrt{64}}$$

$$= \frac{6}{8}$$

$$y'(-6,8) = \frac{3}{4}$$

Implicit Function: A function where y is defined by an *equation in x and y*, such as $x^2 + y^2 = 100$.

If $x^2 + y^2 = 100$ find y' using implicit differentiation.

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = \frac{d}{dx} 100$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \frac{dx}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Find the slope of the circle $x^2 + y^2 = 100$ at the point (6, 8)

$$\left.\frac{dy}{dx}\right|_{(6,8)} = \frac{-(6)}{8}$$

$$= \frac{-3}{4}$$

Find the slope of the circle $x^2 + y^2 = 9$ at the point (-6, 8)

$$\left.\frac{dy}{dx}\right|_{(-6,8)} = \frac{-(-6)}{8}$$

$$= \frac{3}{4}$$

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Ex A: Find each derivative implicitly or explicitly.

#1) $\frac{d}{dx}y^{10} = 10y\frac{dy}{dx}$

#2) $\frac{d}{dx}x^{10} = 10x^9$

#3) $\frac{d}{dx}(x^5y^7) = \frac{d}{dx}x^5 \cdot y^7 + x^5 \frac{d}{dx}y^7$

$= 5x^4y^7 + x^5(7y^6)\frac{dy}{dx}$

$= 5x^4y^7 + 7x^5y^6\frac{dy}{dx}$

#4) $\frac{d}{dx}x = 1$

#5) $\frac{d}{dx}y = \frac{dy}{dx}$

#6) $\frac{d}{dx}(5x^3y^2) = \frac{d}{dx}(5x^3)\,y^2 + 5x^3 \cdot \frac{d}{dx}(y^2)$

$= 15x^2y^2 + 5x^3\, 2y \cdot \frac{dy}{dx}$

$= 15x^2y^2 + 10x^3y\frac{dy}{dx}$

Method for finding dy/dx from an equation that defines y implicitly involves three steps:

1. Differentiate both sides of the equation *with respect to x.*
2. Collect all terms involving $\frac{dy}{dx}$ on one side, and all others on the other side.
3. Factor out the $\frac{dy}{dx}$ and solve for it by dividing.

Ex B: Finding and Evaluating an Implicit Derivative

For $x^4 + y^4 - 2x^2y^2 = 10$ find $\frac{dy}{dx}$ and evaluate it at x = 2, y = 1.

$$\frac{d}{dx}x^4 + \frac{d}{dx}y^4 - \frac{d}{dx}(2x^2y^2) = \frac{d}{dx}(10)$$

$$4x^3 + 4y^3\frac{dy}{dx} - \left[\frac{d}{dx}(2x^2)y^2 + 2x^2\frac{d}{dx}(y^2)\right] = 0$$

$$4x^3 + 4y^3\frac{dy}{dx} - \left[4xy^2 + 2x^2 \cdot 2y \cdot \frac{dy}{dx}\right] = 0$$

$$4x^3 + 4y^3\frac{dy}{dx} - 4xy^2 - 4x^2y\frac{dy}{dx} = 0$$

$$4y^3\frac{dy}{dx} - 4x^2y\frac{dy}{dx} = -4x^3 + 4xy^2$$

$$\frac{dy}{dx}(4y^3 - 4x^2y) = -4x^3 + 4xy^2$$

$$\frac{dy}{dx} = \frac{4x(-x^2 + y^2)}{4y^3 - 4x^2y}$$

$$\frac{dy}{dx} = \frac{4x(-x^2 + y^2)}{4y(y^2 - x^2)}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\frac{dy}{dx} = \frac{x}{-y}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{(2)}{-(1)}$$

$$\left.\frac{dy}{dx}\right|_{(2,1)} = -2$$

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Consumer Demand
In economics, a demand equation is the relationship between the price p of an item and the quantity x that consumers will demand at that price. (All prices are in dollars, unless otherwise stated).

Ex C: Interpreting an Implicit Derivative

For the demand equation $x = \sqrt{1900 - p^3}$ find $\frac{dp}{dx}$. Then evaluate it at x = 30, p =10 and interpret your answer.

Implicitly

$$\frac{d}{dx}(x) = \frac{d}{dx}(1900-p^3)^{\frac{1}{2}}$$
$$1 = \frac{1}{2}(1900-p^3)^{-\frac{1}{2}}\frac{d}{dx}(1900-p^3)$$
$$1 = \frac{1}{2\sqrt{1900-p^3}}(-3p^2)\frac{dp}{dx}$$
$$1 = \frac{-3p^2}{2\sqrt{1900-p^3}}\frac{dp}{dx}$$
$$\frac{2\sqrt{1900-p^3}}{-3p^2} = \frac{dp}{dx}$$
$$\left.\frac{dp}{dx}\right|_{(30,10)} = \frac{2\sqrt{1900-(10)^3}}{-3(10)^2}$$
$$= \frac{2\sqrt{1900-1000}}{-3(100)}$$
$$= \frac{2\sqrt{900}}{-300}$$
$$= \frac{2(30)}{-300}$$
$$= \frac{2}{-10}$$
$$\left.\frac{dp}{dx}\right|_{(30,10)} = \frac{1}{-5}$$

Explicitly

$$x = \sqrt{1900-p^3}$$
$$x^2 = 1900-p^3$$
$$x^2-1900 = -p^3$$
$$-x^2+1900 = p^3$$
$$(-x^2+1900)^{\frac{1}{3}} = p$$
$$\frac{d}{dx}(-x^2+1900)^{\frac{1}{3}} = \frac{dp}{dx}$$
$$\frac{1}{3}(-x^2+1900)^{-\frac{2}{3}}\frac{d}{dx}(-x^2+1900) = \frac{dp}{dx}$$
$$\frac{1}{3\left(\sqrt[3]{-x^2+1900}\right)^2}(-2x) = \frac{dp}{dx}$$
$$\frac{-2x}{3\left(\sqrt[3]{-x^2+1900}\right)^2} = \frac{dp}{dx}$$
$$\frac{-2(30)}{3\left(\sqrt[3]{-(30)^2+1900}\right)^2} = \left.\frac{dp}{dx}\right|_{(30,10)}$$
$$\frac{-60}{3\left(\sqrt[3]{-900+1900}\right)^2} =$$
$$\frac{-20}{\left(\sqrt[3]{1000}\right)^2} =$$
$$\frac{-20}{(10)^2} =$$
$$\frac{-20}{100} =$$
$$-\frac{1}{5} = \left.\frac{dp}{dx}\right|_{(30,10)}$$

With intelligence

$$x = \sqrt{1900-p^3}$$
$$x^2 = 1900-p^3$$
$$\frac{d}{dx}(x^2) = \frac{d}{dx}(1900) - \frac{d}{dx}(p^3)$$
$$2x = -3p^2\frac{dp}{dx}$$
$$\frac{2x}{-3p^2} = \frac{dp}{dx}$$
$$\frac{2(30)}{-3(10)^2} = \left.\frac{dp}{dx}\right|_{(30,10)}$$
$$\frac{2(\cancel{30})^{10}}{-\cancel{3}(\cancel{100})_{10}} =$$
$$\frac{1}{-5} = \left.\frac{dp}{dx}\right|_{(30,10)}$$

When 30 items have been sold at \$10 per item, a \$1 increase in price will result in a decrease of 5 sales.