Derivative Applications
3.4 – Distance, Velocity, Acceleration & Other Stuff

Velocity is the derivative of distance traveled. Acceleration is the second derivative of distance traveled

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$$s(t) = distance\ at\ time\ t$$
$$v(t) = s'(t) = velocity\ at\ time\ t$$
$$a(t) = v'(t) = s''(t) = accerlation\ at\ time\ t$$

Note: Velocity always implies a direction.

Ex A: Distance, Velocity and Acceleration Applications

#1) A delivery truck is driving along a straight road, and after t hours its distance (in miles) east of its starting point is $s(t) = 24t^2 - 4t^3$ for $0 \le t \le 6$.

a. Find the velocity of the truck after 2 hours.

$v(t) = 48t - 12t^2$

$v(2) = 48(2) - 12(2)^2$
$= 96 - 12(4)$
$= 96 - 48$
$v(2) = 48$ mph EAST

After 2 hours the travel distance of the truck is increasing by 48 miles per hour east.

b. Find the velocity of the truck after 5 hours.

$v(5) = 48(5) - 12(5)^2$
$= 240 - 12(25)$
$= 240 - 300$
$= -60$ mph East
$v(5) = 60$ mph west

After 5 hours the truck's travel distance is increasing by 60 miles per hour west.

c. Find the acceleration of the truck after 1 hour.

$a(t) = 48 - 24t$

$a(1) = 48 - 24(1)$
$a(1) = 24$ mph/h

After 2 hours the truck's velocity is increasing by 24 miles per hour each hour.

#2) A helicopter rises vertically and after t second its height above the ground is $s(t) = 6t^2 - t^3$ feet ($0 \le t \le 6$).

a. Find the velocity after 2 seconds.

$v(t) = 12t - 3t^2$

$v(2) = 12(2) - 3(2)^2$
$= 24 - 3(4)$
$= 24 - 12$
$v(2) = 12$ ft/sec up

After 2 seconds the travel distance of the helicopter is increasing by 12 feet per second and is going up.

b. Find the velocity after 5 seconds.

$v(5) = 12(5) - 3(5)^2$
$= 60 - 3(25)$
$= 60 - 75$
$= -15$ ft/sec up
$v(5) = 15$ ft/sec down

After 5 seconds the helicopter's travel distance is increasing by 15 feet per second and is going down.

c. Find the acceleration after 1 second.

$a(t) = 12 - 6t$

$a(1) = 12 - 6(1)$
$= 12 - 6$
$a(1) = 6$ ft/sec/sec

After 1 seconds the helicopter's velocity is increasing by 6 feet per second every second.

Other Applications of Second Derivative

The second derivative applies to many things in addition to acceleration. In general, the second derivative measures how the rate of change is changing. In other words, the second derivative describes whether the rate is speeding up or slowing down.

Ex B: Applications of second derivatives.
A demographer is one who studies the characteristics of human populations. Hugo, the demographer, predict that *t* years from now the population of a city will be $P(t) = 1{,}000{,}000 + 28{,}800t^{1/3}$.

a. Find P(8) and interpret your answer.

$$P(8) = 1{,}000{,}000 + 28{,}800\sqrt[3]{8}$$
$$= 1{,}000{,}000 + 28{,}800(2)$$
$$= 1{,}000{,}000 + 57{,}600$$
$$P(8) = 1{,}057{,}600 \text{ people}$$

Eight years from now the population of a city will be 1,057,600 people

b. Find P'(8) and interpret your answer.

$$P'(t) = 9600t^{-2/3}$$

$$P'(8) = \frac{9600}{(\sqrt[3]{(8)})^2} = \frac{9600}{(2)^2} = \frac{9600}{4}$$
$$P'(8) = 2400 \text{ people/year}$$

Eight years from now the population of a city will be increasing by 2400 people per year.

c. Find P''(8) and interpret your answer.

$$P''(t) = 6400t^{-5/3}$$

$$P''(8) = \frac{6400}{(\sqrt[3]{(8)})^5} = \frac{6400}{(2)^5} = \frac{6400}{32}$$
$$P''(8) = 200 \text{ people/yr}^2$$

Eight years from now the population growth rate will be increasing by 200 people per year each year