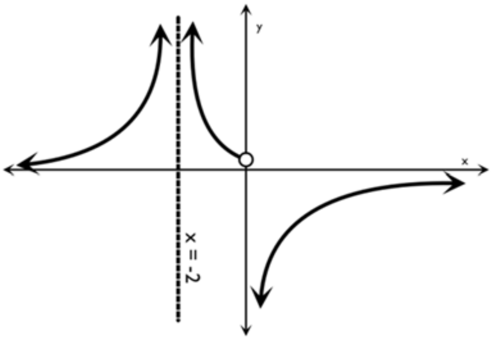
Graphing & Basic Optimization

5.1A – Graphing Using Derivatives

Find the interval for which the derivative is positive and the interval for which the derivative is negative.

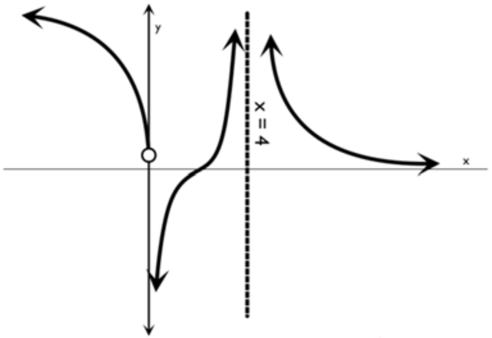
#1)



$f'>0$ when $(-\infty,-2)\cup(0,\infty)$

$f'<0$ when $(-2,0)$

#2)



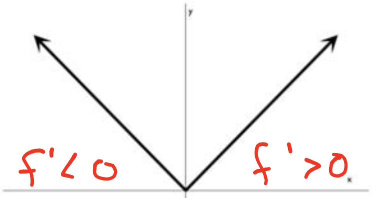
$f'>0$ when $(0,4)$

$f'<0$ when $(-\infty,0)\cup(4,\infty)$

The first column shows graphs of four functions and the second column shows the graphs of their derivatives. Match each function with its derivative.

#3)

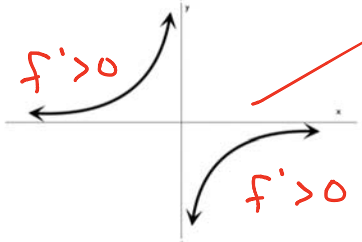
C



$f'<0$ $f'>0$

#4)

A

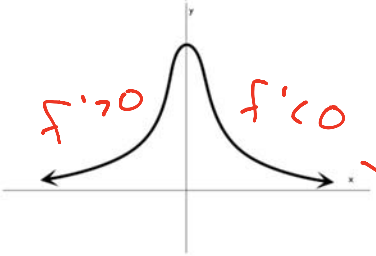


$f'>0$

$f'>0$

#5)

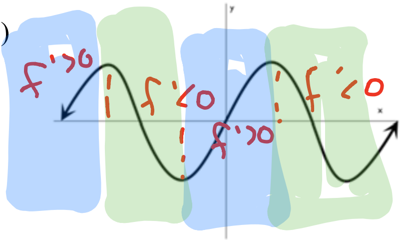
D



$f'>0$ $f'<0$

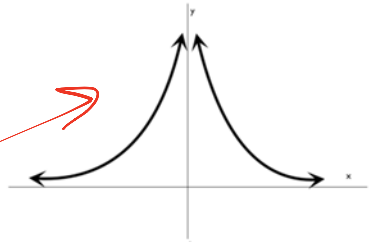
#6)

B

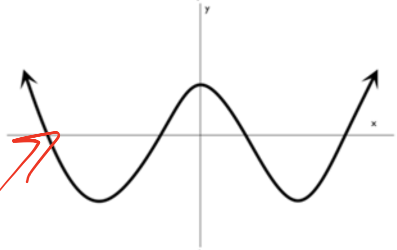


$f'>0$ $f'<0$ $f'>0$ $f'<0$

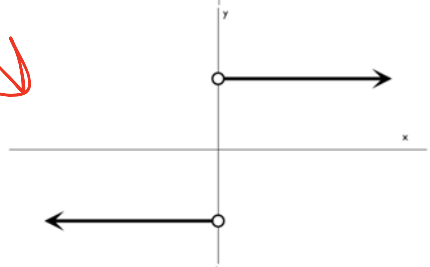
a



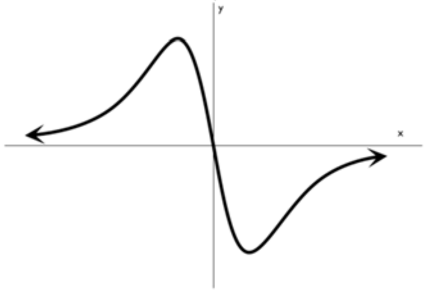
b



c



d



Graphing & Basic Optimization

5.1A – Graphing Using Derivatives

Find the critical values of each function.
(On day 1, just find CV from 1st derivative. On day 2, find CV from 2nd derivative.)

#7) $f(x) = x^3 - 48x$

CV

$f'(x) = 3x^2 - 48$

$0 = 3(x^2 - 16)$

$0 = 3(x-4)(x+4)$

ZON

$0 = 3(x-4)(x+4)$

$0 \neq 3$ } $0 = x-4$, $4 = x$ } $0 = x+4$, $-4 = x$

CV: $x = -4, 4$

CV

$f''(x) = 6x$

$0 = 6x$

ZON

$0 = 6x$

$x = 0$

CV: $x = 0$

#8) $f(x) = x^3 - 6x^2 - 15x + 30$

CV

$f'(x) = 3x^2 - 12x - 15$

$0 = 3(x^2 - 4x - 5)$

$0 = 3(x-5)(x+1)$

ZON

$0 = 3(x-5)(x+1)$

$0 \neq 3$ } $0 = x-5$, $5 = x$ } $0 = x+1$, $-1 = x$

CV: $x = -1, 5$

CV

$f''(x) = 6x - 12$

$0 = 6(x-2)$

ZON

$0 = 6(x-2)$

$0 \neq 6$ } $0 = x-2$, $2 = x$

CV: $x = 2$

#9) $f(x) = x^4 + 4x^3 - 8x^2 + 1$

CV

$f'(x) = 4x^3 + 12x^2 - 16x$

$0 = 4x(x^2 + 3x - 4)$

$0 = 4x(x+4)(x-1)$

Z ON

$0 = 4x(x+4)(x-1)$

$0 = 4x$, $0 = x$ } $0 = x+4$, $-4 = x$ } $0 = x-1$, $1 = x$

CV: $x = -4, 0, 1$

CV

$f''(x) = 12x^2 + 24x - 16$

$0 = 4(3x^2 + 6x - 4)$

ZON

$0 = 4(3x^2 + 6x - 4)$

$0 \neq 4$ } $0 = 3x^2 + 6x - 4$

Doesn't factor. QUAD Formula

$0 \approx (x+2.5)(x-0.5)$

$0 = x + 2.5$, $-2.5 = x$ } $x - 0.5 = 0$, $x = 0.5$

CV: $x = -2.5, 0.5$

#10) $f(x) = (2x - 6)^4$

CV

$f'(x) = 4(2x-6)^3(2)$

$0 = 8(2x-6)^3$

ZON

$0 = 8(2x-6)^3$

$0 \neq 8$ } $0 = (2x-6)^3$

$0 = 2x - 6$

$6 = 2x$

$3 = x$

CV: $x = 3$

CV

$f''(x) = 24(2x-6)^2(2)$

$0 = 48(2x-6)^2$

ZON

$0 = 48(2x-6)^2$

$0 \neq 48$ } $0 = (2x-6)^2$

$0 = 2x - 6$

$6 = 2x$

$3 = x$

CV: $x = 3$

#11) $f(x) = 3x + 5$

CV

$f'(x) = 3$

$0 \neq 3$

CV

$f''(x) = 0$

$0 = 0$

True, so $\mathbb{R}$.

#12) $f(x) = x^3 + x^2 - x + 4$

CV

$f'(x) = 3x^2 + 2x - 1$

$0 = (3x^2 - x) + (3x - 1)$

$0 = x(3x-1) + 1(3x-1)$

$0 = (3x-1)(x+1)$

ZON

$0 = (3x-1)(x+1)$

$0 = 3x - 1$, $1 = 3x$, $\frac{1}{3} = x$ } $0 = x+1$, $-1 = x$

CV: $x = -1, \frac{1}{3}$

CV

$f''(x) = 6x + 2$

$0 = 2(3x+1)$

Z ON

$0 = 2(3x+1)$

$0 = 2$ } $0 = 3x + 1$, $-1 = 3x$, $-\frac{1}{3} = x$

CV: $x = -\frac{1}{3}$

Graphing & Basic Optimization
5.1A – Graphing Using Derivatives

Sketch the graph of each function by hand using a sign diagram. (0n day 1, use first derivative sign diagram. On day two, use the second derivative sign diagram.)

#13) $f(x) = x^3 - 3x^2 - 9x + 10$

①

CV

$f'(x) = 3x^2 - 6x - 9$

$0 = 3(x^2 - 2x - 3)$

$0 = 3(x-3)(x+1)$

ZON

$0 = 3(x-3)(x+1)$

$0 \neq 3$ } $0 = x - 3$ } $0 = x + 1$

$3 = x$ $-1 = x$

CV: $x = -1, 3$

②

CP

$f(-1) = 15$

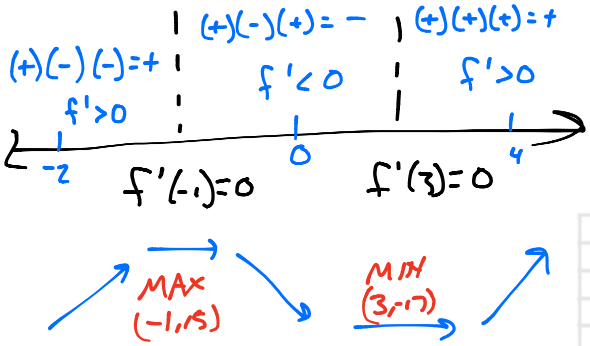
$f(3) = -17$

CP: $(-1,15), (3,-17)$

③

$f'(x) = 3(x-3)(x+1)$

$f'(x) = +(x-3)(x+1)$



④

CV

$f''(x) = 6x - 6$

$0 = 6(x-1)$

ZON

$0 = 6(x-1)$

$0 \neq 6$ } $0 = x - 1$

$1 = x$

CV: $x = 1$

⑤

CP

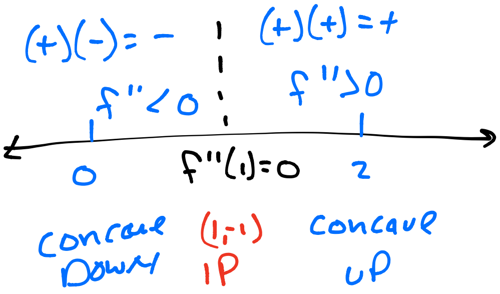
$f(1) = -1$

CP: $(1,-1)$

⑥

$f''(x) = 6(x-1)$

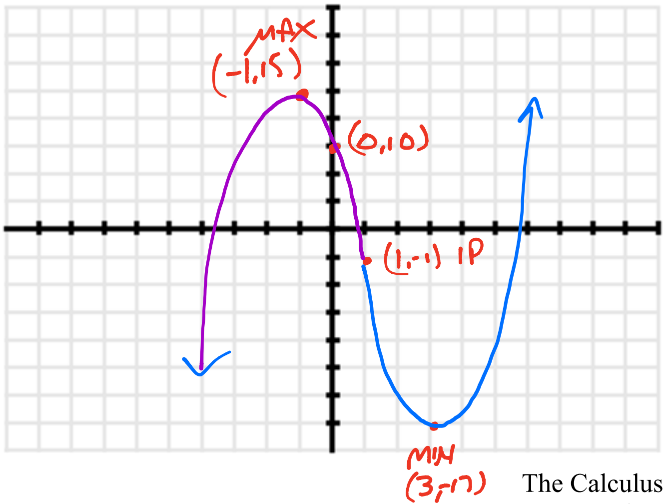
$f''(x) = +(x-1)$



⑦

Y-int

$f(0) = 10$



Graphing & Basic Optimization

5.1A – Graphing Using Derivatives

#14) $f(x) = x^4 + 4x^3 - 8x^2 + 64$

① CV

$f'(x) = 4x^3 + 12x^2 - 16x$

$0 = 4x(x^2 + 3x - 4)$

$0 = 4x(x+4)(x-1)$

ZOH

$0 = 4x(x+4)(x-1)$

$0 = 4x$) $x+4=0$) $x-1=0$

$0 = x$) $x=-4$) $x=1$

CV: $x = -4, 0, 1$

② CP

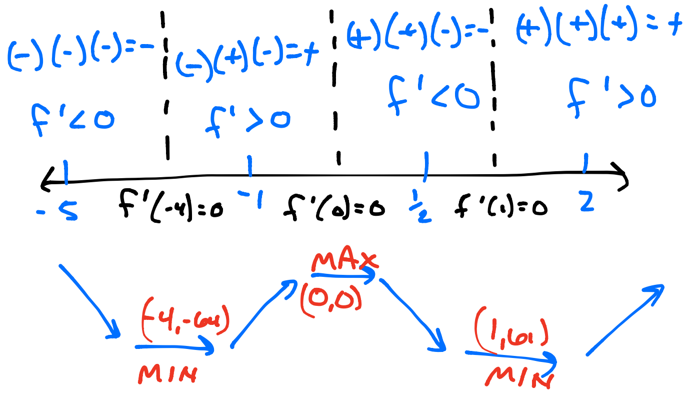
$f(-4) = -64$

$f(0) = 64$

$f(1) = 61$

CP: $(-4,-64), (0,64)(1,61)$

③ $f'(x) = 4x(x+4)(x-1)$



④ CV

$f''(x) = 12x^2 + 24x - 16$

$0 = 4(3x^2 + 6x - 4)$

ZOH

$0 = 4(3x^2 + 6x - 4)$

$0 \neq 4$) $0 = 3x^2 + 6x - 4$

Doesn't factor. QUAD Formula

$0 \approx (x+2.5)(x-0.5)$

$0 = x + 2.5$) $x - 0.5 = 0$

$-2.5 = x$) $x = 0.5$

CV: $x = -2.5, 0.5$

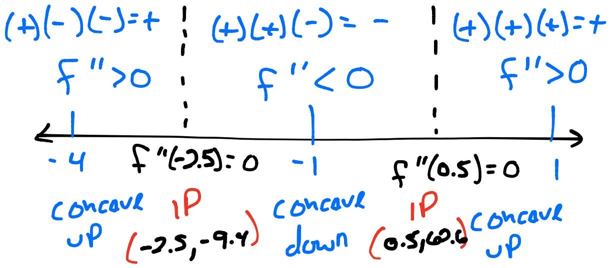
⑤ CP

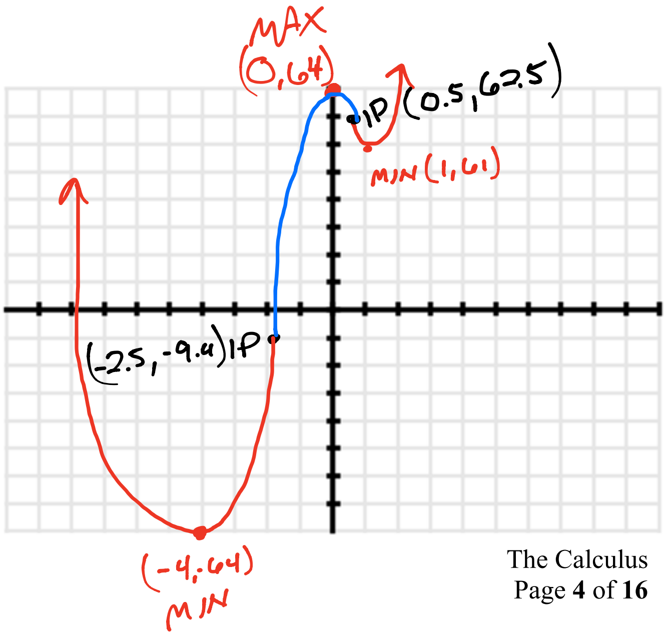
$f(-2.5) = -9.4$

$f(0.5) \approx 62.6$

CP $(-2.5, -9.4), (0.5, 62.6)$

⑥ $f''(x) \approx 4(x+2.5)(x-0.5)$





Graphing & Basic Optimization

5.1A – Graphing Using Derivatives

#15) $f(x) = -x^4 + 4x^3 - 4x^2 + 1$

① CV

$f'(x) = -4x^3 + 12x^2 - 8x$

$0 = -4x(x^2 - 3x + 2)$

$0 = -4x(x-1)(x-2)$

ZON

$0 = -4x(x-1)(x-2)$

$0 = -4x$ | $0 = x-1$ | $0 = x-2$

$0 = x$ | $1 = x$ | $2 = x$

$CV: x = 0, 1, 2$

② CP

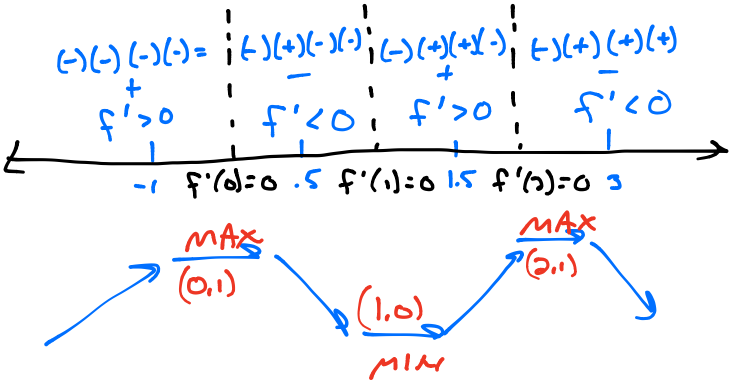
$f(0) = 1$

$f(1) = 0$

$f(2) = 1$

$CP: (0,1), (1,0), (2,1)$

③ $f'(x) = -4x(x-1)(x-2)$



⑦ Y-int

$f(0) = 1$

④ CV

$f''(x) = -12x^2 + 24x - 8$

$0 = -4(3x^2 - 6x + 2)$

ZON

$0 = -4(3x^2 - 6x + 2)$

$0 \neq 4$ | $3x^2 - 6x + 2 = 0$

Doesn't FACTOR. QUAD Form

$(x - 0.4)(x - 1.6) \approx 0$

$x - 0.4 = 0$ | $x - 1.6 \approx 0$

$x = 0.4$ | $x = 1.6$

$CV: x \approx 0.4, 1.6$

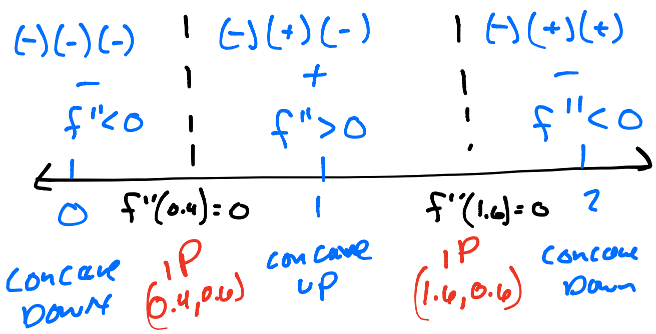
⑤ CP

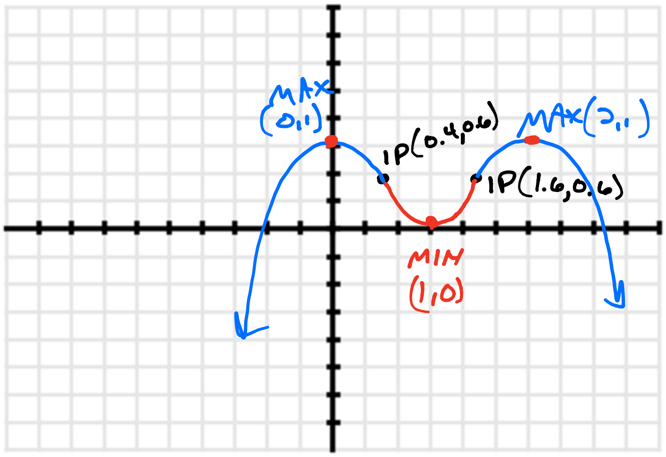
$f(0.4) \approx 0.6$

$f(1.6) \approx 0.6$

$CP: (0.4, 0.6), (1.6, 0.6)$

⑥ $f''(x) \approx -4(x - 0.4)(x - 1.6)$





Graphing & Basic Optimization

5.1A – Graphing Using Derivatives

#16) $f(x) = 3x^4 - 8x^3 + 6x^2$

① CV

$f'(x) = 12x^3 - 24x^2 + 12x$

$0 = 12x(x^2 - 2x + 1)$

$0 = 12x(x-1)^2$

ZON

$0 = 12x(x-1)^2$

$0 = 12x$; $0 = x$

$0 = (x-1)^2$; $0 = x - 1$; $1 = x$

CV: $x = 0, 1$

② CP

$f(0) = 0$

$f(1) = 1$

CP$(0,0), (1,1)$

③ $f'(x) = 12x(x-1)^2$

$(-)(+) = -$	$(+)(+) = +$	$(+)(+) = +$
$f' < 0$	$f' > 0$	$f' > 0$

-1, $f'(0) = 0$, $\frac{1}{2}$, $f'(1) = 0$, 2

$(0,0)$ MIN

Neither $(1,1)$

④ CV

$f''(x) = 36x^2 - 48x + 12$

$0 = 12(3x^2 - 4x + 1)$

$0 = 12(x-1)(3x-1)$

FACTOR

$3x^2 - 4x + 1$

$= 3x^2 \underline{-3x} + \underline{-x} + 1$

$= 3x(x-1) + -1(x-1)$

$= (x-1)(3x-1)$

ZON

$0 = 12(x-1)(3x-1)$

$0 \neq 12$

$0 = x - 1$; $1 = x$

$0 = 3x - 1$; $1 = 3x$; $\frac{1}{3} = x$

CV: $x = \frac{1}{3}, 1$

⑤ CP

$f(\frac{1}{3}) = 0.4$

$f(1) = 1$

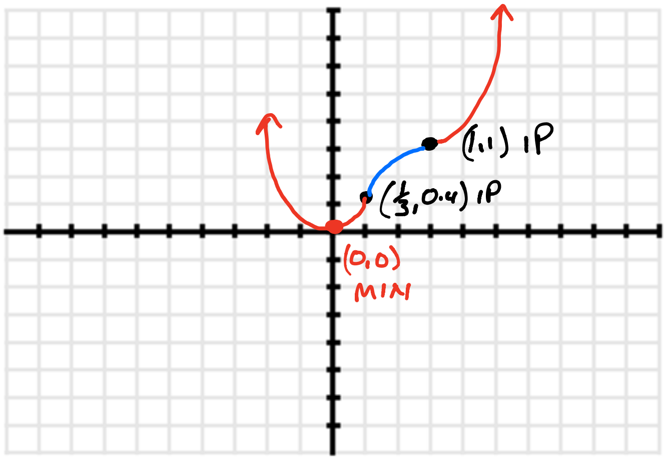
CP $(\frac{1}{3}, 0.4), (1,1)$

⑥ $f''(x) = 12(x-1)(3x-1)$

$(+)(-)(-) = +$	$(+)(-)(+) = -$	$(+)(+)(+) = +$
$f'' > 0$	$f'' < 0$	$f'' > 0$

0, $f''(\frac{1}{3}) = 0$, $.9$, $f''(1) = 0$, 2

concave up, IP $(\frac{1}{3}, 0.4)$, Concave DN, IP $(1,1)$, Concave up



⑦ y-int

$f(0) = 0$

Graphing & Basic Optimization

5.1A – Graphing Using Derivatives

#17) $f(x) = (x - 1)^6$

① CV

$f'(x) = 6(x-1)^5(1)$

$0 = 6(x-1)^5$

ZON

$0 = 6(x-1)^5$

$0 \neq 6$

$0 = (x-1)^5$

$0 = x-1$

$1 = x$

Cv: x = 1

② CP

$f(1) = 0$

CP (1,0)

③ $f'(x) = 6(x-1)^5$

$(+)(-) = -$ | $(+)(+) = +$

$f' < 0$ | $f' > 0$

0 | $f'(1) = 0$ | 2

↘ (1,0) ↗

MIN

④ CV

$f''(x) = 30(x-1)^4(1)$

$0 = 30(x-1)^4$

ZON

$0 = 30(x-1)^4$

$0 = 30$

$0 = (x-1)^4$

$0 = x-1$

$1 = x$

Cv: x = 1

⑤ CP

$f(1) = 0$

CP (1,0)

⑥ $f''(x) = 30(x-1)^4$

$(+)(+) = +$ | $(+)(+) = +$

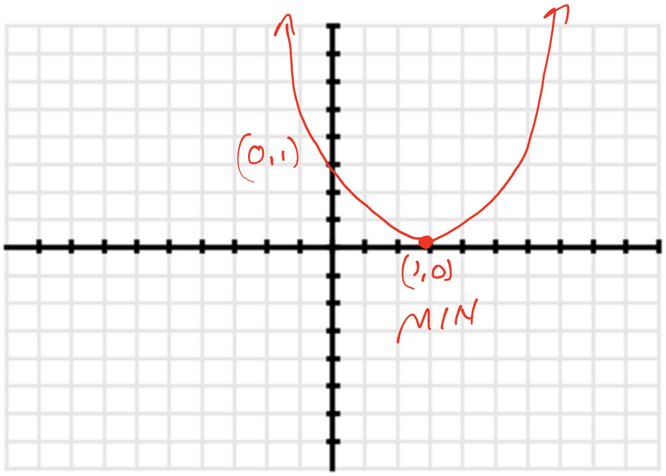
$f'' > 0$ | $f'' > 0$

$f''(1) = 0$

concave up | nothing | concave up

⑦ y-int

$f(0) = 1$



Graphing & Basic Optimization

5.1A – Graphing Using Derivatives

#18) $f(x) = (x^2 - 4)^2$

① CV

$f'(x) = 2(x^2-4)\cdot(2x)$

$0 = 4x(x^2-4)$

$0 = 4x(x-2)(x+2)$

ZON

$0 = 4x(x-2)(x+2)$

$0 = 4x$, $0 = x$; $0 = x-2$, $2 = x$; $0 = x+2$, $-2 = x$

CV: $x = -2, 0, 2$

② CP

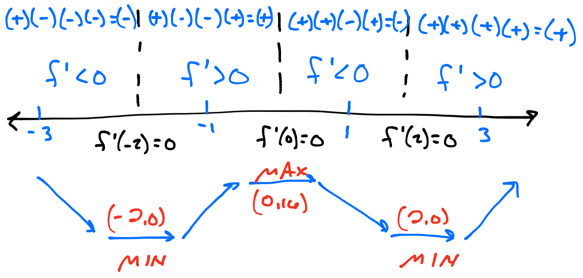
$f(-2) = 0$

$f(0) = 16$

$f(2) = 0$

CP $(-2,0), (0,16), (2,0)$

③ $f'(x) = 4x(x-2)(x+2)$



④ CV

$f'(x) = 4x^3 - 16x$

$f''(x) = 12x^2 - 16$

$0 = 4(3x^2-4)$

ZON

$0 = 4(3x^2-4)$

$0 \neq 4$; $0 = 3x^2 - 4$

$4 = 3x^2$

$\frac{4}{3} = x^2$

$\pm\sqrt{\frac{4}{3}} = x$

CV: $x = -\sqrt{\frac{4}{3}}, \sqrt{\frac{4}{3}}$

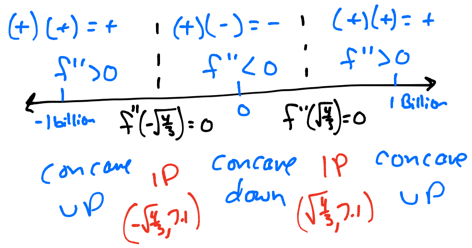
⑤ CP

$f(-\sqrt{\frac{4}{3}}) \simeq 7.1$

$f(\sqrt{\frac{4}{3}}) \simeq 7.1$

CP $(-\sqrt{\frac{4}{3}}, 7.1), (\sqrt{\frac{4}{3}}, 7.1)$

⑥ $f''(x) = 4(3x^2-4)$



⑦ Y-int

$f(0) = 16$