Graphing & Basic Optimization

5.2A – Optimization

A: Find the absolute extreme values of each function on the interval given.

#1) $f(x) = x^3 - 6x^2 + 9x + 10$ on [-1, 2]

CV

① $f'(x) = 3x^2 - 12x + 9$

$0 = 3(x^2 - 4x + 3)$

$0 = 3(x-3)(x-1)$

$0 \neq 3$; $0 = x - 3$, $3 = x$; $0 = x - 1$, $1 = x$

CV: $x = 1, \cancel{3}$ $\quad 3 \notin [-1, 2]$

② EV: $x = -1, 2$

③ $f(1) = 14$ MAX

$f(-1) = -6$ MIN

$f(2) = 12$

CP: $(1, 14)$

EP: $(-1, -6), (2, 12)$

#2) $f(x) = -x + 7$ on [0, 7]

CV

① $f'(x) = -1$

$0 \neq -1$

② EV: $x = 0, 7$

③ $f(0) = 7$ MAX

$f(7) = 0$ MIN

EP: $(0, 7), (7, 0)$

#3) $f(x) = 4x^2 - x^3$ on [0, 6]

CV

① $f'(x) = 8x - 3x^2$

$0 = x(8 - 3x)$

$0 = x$; $0 = 8 - 3x$, $3x = 8$, $x = 8/3$

CV: $x = 0, 8/3$

② EV: $x = 0, 6$

③ $f(0) = 0$

$f(8/3) = 9.5$ MAX

$f(6) = -72$ MIN

CP: $(0, 0), (8/3, 9.5)$

EP: $(0, 0), (6, -72)$

#4) $f(x) = x^3 - 12x$ on [-2, 2]

CV

① $f'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$0 \neq 3$; $0 = x^2 - 4$, $4 = x^2$, $\pm 2 = x$

CV: $x = -2, 2$

② EV: $x = -2, 2$

③ $f(-2) = 16$ MAX

$f(2) = -16$ MIN

CP: $(-2, 16), (2, -16)$

EP: $(-2, 16), (2, -16)$

Graphing & Basic Optimization
5.2A – Optimization

A: Find the absolute extreme values of each function on the interval given.

#5) $f(x) = x^4 + 2x^3 + 2x^2 - 1$ on [-2, 2]

CV

1. $f'(x) = 4x^3 + 6x^2 + 4x$
   $0 = 2x(2x^2 + 3x + 2)$
   $0 = 2x$, $0 = x$; $0 = 2x^2 + 3x + 2$ Doesn't factor; Quad Formula $x =$ dne.
   CV: $x = 0$
2. EV: $x = -2, 2$
3. $f(0) = -1$ MIN
   $f(-2) = 7$
   $f(2) = 39$ MAX
   CP: $(0, -1)$
   EP: $(-2, 7), (2, 39)$

#6) $f(x) = 2x^5 - 3x^4$ on [-1, 4]

CV

1. $f'(x) = 10x^4 - 12x^3$
   $0 = 2x^3(5x - 6)$
   $0 = 2x^3$, $0 = x^3$, $0 = x$; $0 = 5x - 6$, $6 = 5x$, $6/5 = x$
   CV: $x = 0, 6/5$
2. EV: $x = -1, 4$
3. $f(0) = 0$
   $f(6/5) = -1.2$
   $f(-1) = -5$ MIN
   $f(4) = 1280$ MAX
   CP: $(0, 0), (6/5, -1.2)$
   EP: $(-1, -5), (4, 1280)$

#7) $f(x) = (x^2 - 1)^3$ on [-1, 1]

CV

1. $f'(x) = 3(x^2 - 1)^2 (x^2 - 1)'$
   $f'(x) = 3(x^2 - 1)^2 (2x)$
   $0 = 6x(x^2 - 1)^2$
   $0 = 6x$, $0 = x$; $0 = (x^2 - 1)^2$, $0 = x^2 - 1$, $1 = x^2$, $\pm 1 = x$
   CV: $x = -1, 0, 1$
2. EV: $x = -1, 1$
3. $f(-1) = 0$ MAX
   $f(0) = -1$ MIN
   $f(1) = 0$ MAX
   CP: $(-1, 0), (0, -1), (1, 0)$
   EP: $(-1, 0), (1, 0)$

#8) $f(x) = \frac{x}{x^2+1}$ on [-4, 2]

$f'(x) = \frac{(x)'(x^2+1) - x(x^2+1)'}{(x^2+1)^2}$

$= \frac{(1)(x^2+1) - x(2x)}{(x^2+1)^2}$

$= \frac{x^2 + 1 - 2x^2}{(x^2+1)^2}$

$f'(x) = \frac{1 - x^2}{(x^2+1)^2}$

$0 = 1 - x^2$, $x^2 = 1$, $x = \pm 1$; $0 = (x^2+1)^2$, $0 = x^2 + 1$, $-1 = x^2$, $\pm\sqrt{-1} = x$, und $= x$

CV: $x = \pm 1$

EV: $x = -4, 2$

3. $f(-1) = -\frac{1}{2}$ MIN
   $f(1) = \frac{1}{2}$ MAX
   $f(-4) = -.24$
   $f(2) = .4$
   CP: $(-1, -\frac{1}{2}), (1, \frac{1}{2})$
   EP: $(-4, -0.24), (2, .4)$

Parasites
#9) George has a scorching case of parasites. The average parasite count living on him on day x of his unbathing season is $P(x) = 8x - 0.2x^2$ (for 0 < x < 40). On which day is the parasite count the highest?

$P(x)$ = # of parasites
x = days of unbathing

CV

① $P'(x) = 8 - 0.4x$

$0 = 8 - 0.4x$

$0.4x = 8$

CV: $x = 20$

② $P''(x) = -0.4$

$P''(20)$ = neg, concave down, MAX

③ $P(20) = 80$

CP: (20, 80)

Sentence Answer:
Twenty days into his unbathing season, George's parasites will reach a maximum number of 80.

Moped (pronounced Moe Ped)
#10) The fuel economy (in miles per gallon) of George's Moped is $E(x) = -0.01x^2 + 0.62x + 10.4$, where x is the driving speed (in miles per hour, $20 \le x \le 60$). At what speed is fuel economy greatest?

E = mpg
x = speed mph

CV

① $E'(x) = -0.02x + .62$

$0 = -0.02x + .62$

$0.02x = .62$

CV: $x = 31$

② EV: $x = 20, 60$

③ $E(31) = 20.01$ MAX

$E(20) = 18.8$

$E(60) = 11.6$

CP: (31, 20.01)

EP: (20, 18.8), (60, 11.6)

Sentence Answer:
The fuel economy is maximized at 31 mph, giving about 20 mpg

Toxic Waste

#11) George's body is discharging toxic waste into a large lake, and the pollution level at a point x miles from George's disgustingness is $P(x) = 3x^2 - 72x + 576$ parts per million ($0 \le x \le 50$). Find where the pollution is the least.

P = pollution in parts per million
x = miles

$P'(x) = 6x - 72$
$0 = 6x - 72$
$72 = 6x$
CV: $12 = x$

EV: $x = 0, 50$

$P(12) = 144$ MIN
$P(0) = 576$
$P(50) = 4476$

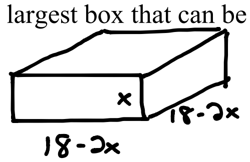
CP: $(12, 144)$
EP: $(0, 576), (50, 4476)$

Sentence Answer:

Twelve miles from George's disgustingness, the pollution is the least at 144 parts per million.

Cabbage Patch

#12) George was finally able to sell his vintage, extremely used Cabbage Patch Doll on Ebay for the high price of 14 cents. To keep his profit margin as high as possible, George is making his own open top packing box from a square piece of cardboard with dimensions of 18 inches. (He plans to use toilet paper for the lid.) If George cuts the corners out of the cardboard, what are the dimensions and volume of the largest box that can be made this way?



x = length of cut
V = volume
Domain = (0, 9)

$V = \text{Area}_{base} \cdot \text{height}$
$V = (18-2x)(18-2x)x$
$V = (324 - 72x + 4x^2)x$
$V = 324x - 72x^2 + 4x^3$

$V' = 324 - 144x + 12x^2$
$V' = 12(x^2 - 12x + 27)$
$0 = 12(x-3)(x-9)$
$0 \ne 12$ | $0 = x-3$, $3 = x$ | $0 = x-9$, $9 = x$

CV: $x = 3, 9$ $9 \notin$ Domain

$V'' = 24x - 144$
$V'' = 24(x-6)$
$V''(3) = (+)(-) = \text{neg}$, concave up
MAX

length $= 18 - 2x$
$= 18 - 2(3)$
$= 18 - 6$
$= 12$

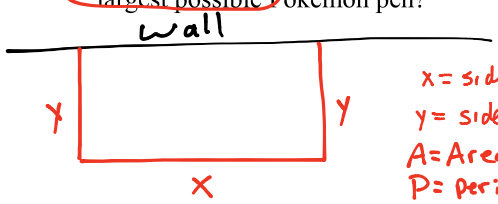
$V(3) = 432 \text{ in}^3$

Sentence Answer:

George should cut 3" from corners, giving dimensions of 12"x12"x3" and a maximum volume of 432 in^3.

Pocket Monster Pen

#13) George wants to build a Pokemon pen along the side of his bedroom wall using 800 inches of popsicle sticks. If the side of along the bedroom wall needs no popsicle sticks, what are the dimensions of the largest possible Pokemon pen?



x = side // to wall (inches)
y = side ⊥ to wall (inches)
A = Area
P = perimeter

$P = x + y + y$

$800 = x + 2y$

$800 - 2y = x$

$800 - 2(200) = x$

$800 - 400 = x$

$400 = x$

$A = xy$

$A = (800 - 2y)y$

$A = 800y - 2y^2$

$A' = 800 - 4y$

$0 = 800 - 4y$

$4y = 800$

$y = 200$

$A'' = -4$

$A''(200) =$ neg, concave Down, MAx

$A(400, 200) = (400)(200)$

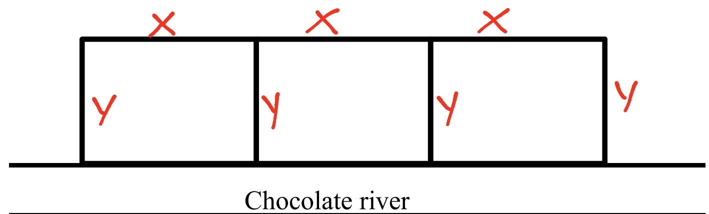
$A(400, 200) = 80{,}000$ inches^2

Sentence Answer:

The largest possible area for George's Pokemon pen is $80{,}000 \text{ in}^2$, with the side parallel to the wall is 400 inches and the sides ⊥ are 200 inches.

Chocolate River

#14) George wants to make three identical rectangular enclosures along a chocolate river for his marshmallow farm, as in the diagram shown below. If he has 1200 inches of pretzel fence (and if the sides along the chocolate river needs no pretzels), what should be the dimensions of each enclosure if the total area is to be maximized?



x = // side to River for each enclosure inches
y = ⊥ side to River for each enclosure inches

$P = 3x + 4y$

$1200 = 3x + 4y$

$1200 - 3x = 4y$

$300 - \frac{3}{4}x = y$

$A = (3x)(y)$

$A = 3xy$

$A = 3x\left(300 - \frac{3}{4}x\right)$

$A = 900x - \frac{9}{4}x^2$

$A' = 900 - \frac{9}{2}x$

$0 = 900 - \frac{9}{2}x$

$\frac{9}{2}x = 900$

$\frac{1}{2}x = 100$

$x = 200$

$300 - \frac{3}{4}(200) = y$

$300 - 3(50) = y$

$300 - 150 = y$

$150 = y$

$A'' = -\frac{9}{2}$

$A''(200) =$ neg, CCD, MAx

$A(200, 150) = (200)(150)$

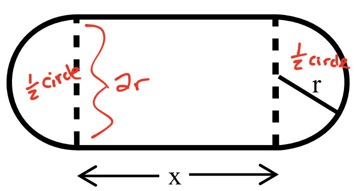
$A(200, 150) = 30{,}000$

Sentence Answer:

To max the area, each enclosure is 200 inches (// to river) by 150 inches (⊥ to river) giving a max area of $30{,}000 \text{ in}^2$ per enclosure

Body Odor

#15) George's B.O. medicine comes in a capsule consisting of a rectangle with a semicircle at each end as shown below. If the perimeter off a cross section is exactly 440 centimeters, find the dimensions (x and r) that maximize the area of the **rectangle**.



$C_{\circ} = 2\pi r$

$A_{Rec} = (2r)x$

$A = 2r(200 - \pi r)$

$A = 440r - 2\pi r^2$

$A' = 440 - 4\pi r$

$0 = 440 - 4\pi r$

$4\pi r = 440$

$r = \frac{110}{\pi}$

$r \approx 35$ cm

$P = 2x + 2\pi r$

$440 = 2x + 2\pi r$

$440 - 2\pi r = 2x$

$220 - \pi r = x$

$220 - \pi(35) = x$

$220 - 35\pi \approx x$

$110 \approx x$

$A'' = -2\pi$

$A''(35) = $ neg, CCD, MAX

$A(35, 110) = 2(35)(110)$

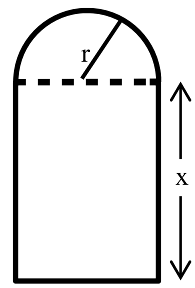
$= 7700 \text{ cm}^2$

Sentence Answer:

To maximize the area of the rectangle to 7700 cm², r is about 35 cm and x is about 110cm

Window

#16) George's bedroom window consists of a rectangle topped by a semicircle, as show below. If the perimeter is to be 18 feet, find the dimensions (x and r) that maximize the area of the window.



$A_{Window} = \frac{1}{2}A_{\circ} + A_{Rec}$

$A = \frac{1}{2}(2\pi r) + x(2r)$

$A = \pi r + 2rx$

$A = \pi r + 2r(9 - r - \frac{\pi}{2}r)$

$A = \pi r + 18r - 2r^2 - \pi r^2$

$A' = \pi + 18 - 4r - 2\pi r$

$0 = \pi + 18 - 4r - 2\pi r$

$-\pi - 18 = -4r - 2\pi r$

$-\pi - 18 = r(-4 - 2\pi)$

$\frac{-\pi - 18}{-4 - 2\pi} = r$

$2 \approx r$

$P = 2r + x + x + \frac{1}{2}(2\pi r)$

$18 = 2r + 2x + \pi r$

$18 - 2r - \pi r = 2x$

$9 - r - \frac{\pi}{2}r = x$

$9 - (2) - \frac{\pi}{2}(2) = x$

$7 - \pi \approx x$

$3.9 \approx x$

$A = \pi(2) + 2(2)(3.9)$

$A = 2\pi + 15.6$

$A \approx 21.9 \text{ ft}^2$

$A'' = -4 - 2\pi$

$A''(2) = $ neg, CCD, MAX

Sentence Answer:

To maximize the area of the window to 22.9 ft², r is about 2ft and x is about 3.9 ft.