Graphing & Basic Optimizing
5.3A – Maximizing Profit

A: Find the absolute extreme values of each function on the interval given.

#1) $f(x) = x^3 - 6x^2 + 9$ on [-3, 3]

$f'(x) = 3x^2 - 12x$

$0 = 3x(x-4)$

$0 = 3x \quad 0 = x-4$

$0 = x \quad 4 = x$

CV: $x = 0, \cancel{4}$ $\quad 4 \notin [-3,3]$

EV: $x = -3, 3$

$f(0) = 9$ MAX

$f(-3) = -72$ MIN

$f(3) = -18$

CP: $(0, 9)$

EP: $(-3, -72)$, $(3, -18)$

MAX $(0, 9)$

MIN $(-3, -72)$

#2) $f(x) = x(x - 10)$ on [-10, 10]

$f(x) = x^2 - 10x$

$f'(x) = 2x - 10$

$0 = 2x - 10$

$10 = 2x$

$5 = x$

CV: $x = 5$

EV: $x = -10, 10$

$f(5) = -25$ MIN

$f(-10) = 200$ MAX

$f(10) = 0$

CP: $(5, -25)$

EP: $(-10, 200)$, $(10, 0)$

MAX $(-10, 200)$

MIN $(5, -25)$

#3) $f(x) = \sqrt[3]{x^2}$ on [-2, 10]

$f(x) = x^{2/3}$

$f'(x) = \frac{2}{3} x^{-1/3}$

$0 = \frac{2}{3\sqrt[3]{x}}$

Z.O.N
$0 \neq 2$

Z.O.D.
$0 = 3\sqrt[3]{x}$
$0 = \sqrt[3]{x}$
CV: $0 = x$

EV: $x = -2, 10$

$f(0) = 0$ MIN
$f(-2) = 1.6$
$f(10) = 4.6$ MAX

CP: (0, 0)
EP: (10, 4.6), (-2, 1.6)

MAX (10, 4.6)
MIN (0, 0)

#4) $f(x) = \frac{1}{x^2+4}$ on [-4, 4]

$f(x) = (x^2+4)^{-1}$

$f'(x) = -1(x^2+4)^{-2} \cdot (x^2+4)'$

$0 = \frac{-2x}{(x^2+4)^2}$

ZON
$0 = -2x$
$0 = x$

ZOD
$0 = (x^2+4)^2$
$0 = x^2+4$
$-4 = x^2$
$\pm\sqrt{-4} = x$
$x =$ dne

CV: $x = 0$

EV: $x = -4, 4$

$f(0) = .25$ MAX
$f(-4) = .05$ MIN
$f(4) = .05$ MIN

CP: (0, 0.25)
EP: (-4, 0.05), (4, 0.05)

MAX (0, 0.25)
MIN (-4, 0.05), (4, 0.05)

Graphing & Basic Optimizing
5.3A – Maximizing Profit

Jimmy Crack Corn

#5) Jimmy Crack Corn finds that it costs $22 to crack each corn, and fixed costs are $38 per day. The price function is $p(x) = 40 - 3x$, where p is the price (in dollars) at which exactly x cracks will be sold. Find the quantity Jimmy Crack Corn should produce and the price he should charge to maximize profit. Also find the maximum profit.

$C(x) = 22x + 38$

$R(x) = p(x)\,qty$

$= (40-3x)x$

$R(x) = 40x - 3x^2$

$P(x) = R(x) - C(x)$

$= (40x - 3x^2) - (22x + 38)$

$P(x) = -3x^2 + 18x - 38$

$P'(x) = -6x + 18$

$0 = -6x + 18$

$6x = 18$

$x = 3$

$P''(x) = -6$

$P''(3) = $ neg, CCD, MAX

$P(3) = -3(3)^2 + 18(3) - 38$

$= -3(9) + 54 - 38$

$= -27 + 16$

$P(3) = -\$11$

$p(3) = 40 - 3(3)$

$= 40 - 9$

$p(3) = 31$

Sentence Answer:

In order to maximize his profit, George should sell 3 cracked corns at $31, giving him a max profit of -$11.

Lady McButter Pants

#6) Lady McButter Pants finds that it costs $200 to manufacture each pair of butter paints, and fixed costs are $1500 per day. The price function is $p(x) = 380 - 9x + 13000x^{-1}$, where p is the price (in dollars) at which exactly x pants will be sold. Find the quantity Lady McButter Pants should produce and the price she should charge to maximize profit. Also find the maximum profit.

$C(x) = 200x + 1500$

$R(x) = p(x)\,qty$

$= (380 - 9x + 13000x^{-1})x$

$R(x) = 380x - 9x^2 + 13{,}000$

$P(x) = R(x) - C(x)$

$= (380x - 9x^2 + 13{,}000) - (200x + 1500)$

$P(x) = -9x^2 + 180x + 11500$

$P'(x) = -18x + 180$

$0 = -18x + 180$

$18x = 180$

$x = 10$

$P''(x) = -18$

$P''(10) = $ neg, CCD, MAX

$P(10) = -9(10)^2 + 180(10) + 11500$

$= -9(100) + 1800 + 11500$

$= -900 + 1800 + 11{,}500$

$= 900 + 11{,}500$

$P(10) = \$12{,}400$

$p(10) = 380 - 9(10) + 13000(10)^{-1}$

$= 380 - 90 + 1300$

$= 290 + 1300$

$p(10) = 1590$

Sentence Answer:

In order to max profit at $12,400, Lady McButter Pants should sell 10 pants at $1590 each.

Floppy Inc

#7) Floppy Inc finds that it costs \$85 to manufacture each pair of Flippies. The price function is $p(x) = 481 - 9x + 30x^{-1}$, where p is the price (in dollars) at which exactly x Flippies will be sold. Find the quantity Floppy Inc should produce and the price it should charge to maximize profit. Also find the maximum profit.

$C(x) = 85x$ (Fixed costs are not mentioned)

$R(x) = p(x)\ qty$

$= (481 - 9x + 30x^{-1})x$

$R(x) = -9x^2 + 481x + 30$

$P(x) = R(x) - C(x)$

$= (-9x^2 + 481x + 30) - (85x)$

$= -9x^2 + 396x + 30$

$P'(x) = -18x + 396$

$0 = -18x + 396$

$18x = 396$

$x = 22$

$P''(x) = -18$

$P''(22) = \text{neg, CCD, MAX}$

$P(22) = -9(22)^2 + 396(22) + 30$

$= -9(484) + 8712 + 30$

$= -4356 + 8742$

$P(22) = \$4386$

$p(22) = 481 - 9(22) + 30(22)^{-1}$

$= 481 - 198 + \frac{30}{22}$

$= 283 + \frac{15}{11}$

$p(22) = 284.36$

Sentence Answer:

To max profit at \$4386, Floppy Inc needs to sell 22 Flippies at \$284.36 each

Brick House Company

#8) The Brick House Company finds that it costs \$7 to manufacture each toilet, and fixed costs are \$20,000 per day. The price function is $p(x) = 307 - 2x + 40{,}000x^{-1}$, where p is the price (in dollars) at which exactly x toilets will be sold. Find the quantity the Brick House Company should produce and the price it should charge to maximize profit. Also find the maximum profit.

$C(x) = 7x + 20{,}000$

$R(x) = p(x)\ qty$

$= (307 - 2x + 40{,}000x$

$R(x) = -2x^2 + 307x + 40{,}000$

$P(x) = R(x) - C(x)$

$= (-2x^2 + 307x + 40{,}000) -$

$P(x) = -2x^2 + 300x + 20{,}000$

$P'(x) = -4x + 300$

$0 = -4x + 300$

$4x = 300$

$x = 75$

$P''(x) = -4$

$P''(75) = \text{neg, CCD, MAX}$

$P(75) = -2(75)^2 + 300(75) + 20{,}000$

$= -2(5625) + 22{,}500 + 20{,}000$

$= -11{,}250 + 42{,}500$

$P(75) = \$31{,}250$

$p(75) = 307 - 2x + 40{,}000x^{-1}$

$= 307 - 2(75) + 40{,}000(75)^{-1}$

$= 307 - 150 + \frac{40{,}000}{75}$

$= 157 + \frac{40{,}000}{75}$

$p(75) = 690.33$

Sentence Answer:

To max profit at \$31,250, Brick House needs to sell 75 toilets at \$690.33