Name________________________Date____________Class________

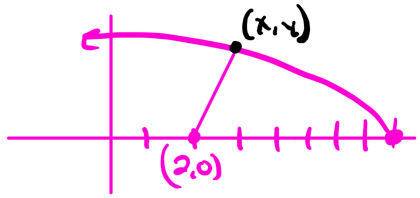
CALC

Homework 5.6

1. Find the point on the graph of $f(x)=\sqrt{-x+8}$ so that the point (2, 0) is closest to the graph.

$-x+8>0$

$8>x$



① $d=\sqrt{[\Delta x]^2+[\Delta y]^2}$

$d=\sqrt{[x-2]^2+[y-0]^2}$

② $\boxed{y=\sqrt{-x+8}}$

③ $d=\sqrt{[x-2]^2+[\sqrt{-x+8}]^2}$

$d=\sqrt{x^2-4x+4+-x+8}$

$d=(x^2-5x+12)^{1/2}$

$d'(x)=0$ when $x=5/2$

d': NEG ($x=0$) | $5/2$ | POS ($x=5$) | 8

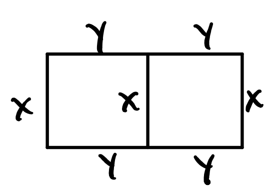
Since d' changes from + to − at $x=5/2$, the distance is a minimum.

$f(5/2)=\sqrt{-5/2+16/2}$

$f(5/2)=\sqrt{\frac{11}{2}}$

⑤ Point: $\left(\frac{5}{2},\sqrt{\frac{11}{2}}\right)$ is the closest to $(2,0)$

2. A rancher has 200 total feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should each corral be so that the enclosed area will be a maximum?



x = length (ft)
y = width (ft)
A = Area (ft²)
F = Fence perimeter

① $A=x\cdot 2y$

③ $A=\cancel{2}x\cdot\left(\frac{200-3x}{\cancel{4}\,2}\right)$

$A=\frac{200x-3x^2}{2}$

$A=100x-\frac{3}{2}x^2$

② $\boxed{\begin{aligned}F&=3x+4y\\ 200&=3x+4y\\ 200-3x&=4y\\ \frac{200-3x}{4}&=y\end{aligned}}$

$\frac{200-3\left(\frac{100}{3}\right)}{4}=y$

$\frac{200-100}{4}=y$

$\frac{100}{4}=y$

$25=y$

④ $A'=100-3x$

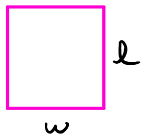
$A'=0$	$A'=$ und
$100-3x=0$	Never
$100=3x$	
$\frac{100}{3}=x$	

CANDIDATE'S TEST

x	$A(x)$
0	0
100/3	1666.667
200/3	0

⑤ The dimension of each corrals should be 25 feet by $\frac{100}{3}$ feet where the shared side is $\frac{100}{3}$ feet.

3. The area of a rectangle is 64 square feet. What dimensions of the rectangle would give the smallest perimeter?



① $P = 2w + 2y$

② $\boxed{\begin{array}{l} A = wl \\ 64 = wl \\ \frac{64}{w} = l \end{array}}$ → $\frac{64}{8} = l$, $8 = l$

③ $P = 2w + 2\left(\frac{64}{w}\right)$

$P = 2w + 128w^{-1}$

④ $P' = 2 - 128w^{-2}$

$P' = \frac{2w^2 - 128}{w^2}$

$P' = \frac{2(w-8)(w+8)}{w^2}$

$P' = 0$		$P' = \text{und}$
$w - 8 = 0$	$w + 8 = 0$	$w^2 = 0$
		$w = 0$
$w = \cancel{-8}, 8$		

NEG POS

P': 0 ——(x=1)—— 8 ——(x=100)——→

P' changes from − to + at $x = 8$, so perimeter is MIN.

⑤ The rectangle should have dimensions 8 feet by 8 feet.

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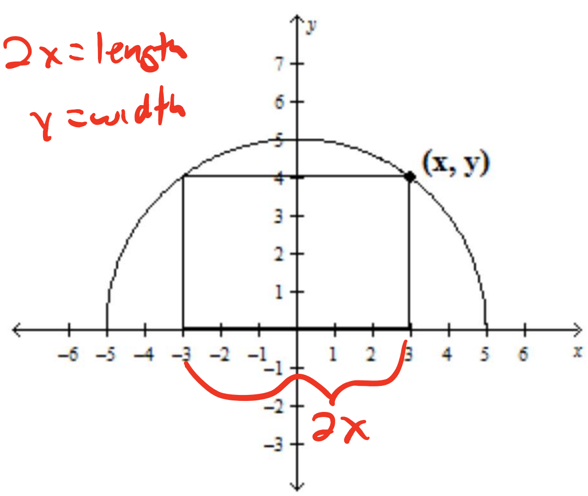
4. A rectangle is bound by the x – axis and the graph of a semicircle defined by $\boxed{y = \sqrt{25 - x^2}}$ ②. What length and width should the rectangle have so that its area is a maximum?

$2x = \text{length}$

$y = \text{width}$



① $A = 2xy$

③ $A = 2x(25 - x^2)^{1/2}$

④ $A'(x) = 0$ at $x = -3.536$, $x = 3.536$ } Same answer in Context

CANDIDATES

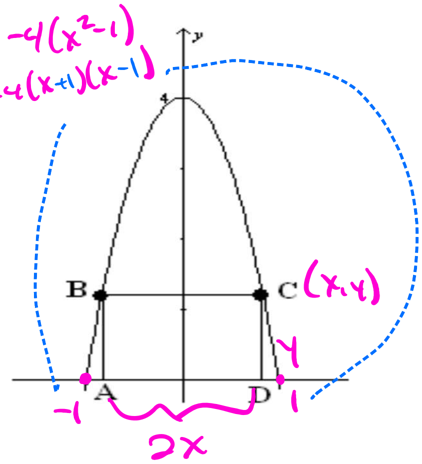
x	$A(x)$
0	0
3.536	25
5	0

⑤ The length should be $2(3.536)$ and the width is $\sqrt{25 - (3.536)^2}$ to maximize Area.

5. A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ as shown in the figure below. Find the x and y coordinates of the point C so that the area of the rectangle is a maximum.

$0 = -4(x^2-1)$

$0 = -4(x+1)(x-1)$



① $A = 2xy$

② $y = -4x^2 + 4$

③ $A = 2x(-4x^2+4)$

$A = -8x^3 + 8x$

④ $A' = -24x^2 + 8$

$A' = -24(x^2 - \frac{1}{3})$

$A' = -24(x - \sqrt{\frac{1}{3}})(x + \sqrt{\frac{1}{3}})$

$A' = 0$	$A' =$ und
$0 = -24(x-\sqrt{\frac{1}{3}})(x+\sqrt{\frac{1}{3}})$	never
$x = \cancel{-\sqrt{\frac{1}{3}}}, \sqrt{\frac{1}{3}}$	

$y = -4x^2 + 4$

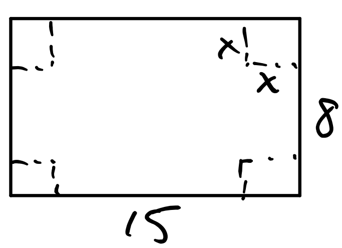
$y = -4(\sqrt{\frac{1}{3}})^2 + 4$

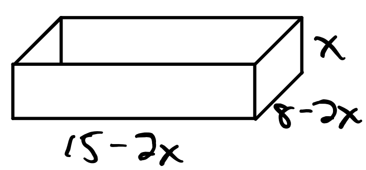
④ A': on $[-1, 1]$, split at $\sqrt{\frac{1}{3}}$: POS (test $x=0$), NEG (test $x=3/4$)

[A' changes from + to − at $x = \sqrt{\frac{1}{3}}$ so Area is maximized.]

⑤ $\boxed{\text{The point } \left(\sqrt{\tfrac{1}{3}}, -4\left(\sqrt{\tfrac{1}{3}}\right)^2 + 4\right) \text{ maximizes the area}}$

6. Find the maximum volume of a box that can be made by cutting squares from the corners of an 8 inch by 15 inch rectangular sheet of cardboard and folding up the sides. $0 < x < 4$





x = length of cut (in)

V = volume

① $V = (15-2x)(8-2x)x$

$V = (120 - 16x - 30x + 4x^2)x$

$V = 120x - 46x^2 + 4x^3$

$V = 4x^3 - 46x^2 + 120x$

④ $V' = 12x^2 - 92x + 120$

$V' = 4(3x^2 - 23x + 30)$

$V' = 4(3x-5)(x-6)$

$\boxed{\begin{array}{l} M = 90x^2 \\ A = -23x \\ N = -5x, -18x \\ 3x^2 - 23x + 30 \\ \underline{3x^2 - 5x} - 18x + 30 \\ x(3x-5) - 6(3x-5) \\ (3x-5)(x-6) \end{array}}$

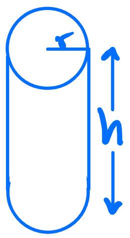
$V' = 0$	$V' =$ und
$x = \cancel{6}, 5/3$	never

V': on $[0, 6]$, split at $5/3$: POS (test $x=1$), NEG (test $x=5$)

[V' changes from + to − at $x = 5/3$ so Volume is maximized.]

⑤ $\boxed{V = [15 - 2(5/3)][8 - 2(5/3)](5/3) \text{ in}^3 \text{ is the maximum volume.}}$

7. The volume of a cylindrical tin can with a top and bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must the height in inches, of the can be?



① r = radius (in)
h = height (in)
V = Volume (in^3)
SA = Surface Area (in^2)

② $V = A_{base} \cdot h$

$16\pi = \pi r^2 \cdot h$

$\frac{16}{r^2} = h$

③ $SA = 2A_{base} + A_{lateral}$

$SA = 2\pi r^2 + 2\pi r \cdot h$

④ $SA = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$

$SA = 2\pi r^2 + 32\pi r^{-1}$

$SA' = 4\pi r - 32\pi r^{-2}$

$SA' = \frac{4\pi r^3 - 32\pi}{r^2}$

$SA' = \frac{4\pi(r^3 - 8)}{r^2}$

⑤

$SA' = 0$	$SA' =$ und
$4\pi(r^3-8) = 0$	$r^2 = 0$
$r^3 - 8 = 0$	$r = 0$
$r^3 = 8$	
$r = 2$	

⑥

	0	NEG	2	POS
SA'		$x=1$		$x=3$

SA' changes from − to + at $x = 2$ so Surface Area is minimized.

⑦ $\frac{16}{2^2} = h$

$\frac{16}{4} = h$

$4 = h$

The height must be 4 inches to minimize amount of tin.