Calculus

Write your questions and thoughts here!

1.3 Asymptotes

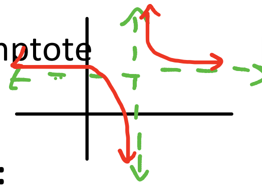
Name:____________________________

Notes

Recall: Vertical Asymptote

Forces graph to + or $-\infty$

(Graph can't cross)



Horizontal Asymptote

Describes end behavior

(graph can cross)

Vertical Asymptotes:

True or False. If you have the function $f(x) = \frac{blah,\ blah,\ blah}{x-a}$ then there must be a vertical asymptote at $x = a$. False, there could be a hole @ $x=a$

Use the function $f(x) = \frac{x^2+2x-8}{x^2+x-12}$ to answer the following.

1. Identify all vertical asymptotes.

$f(x) = \frac{\cancel{(x+4)}(x-2)}{\cancel{(x+4)}(x-3)} = \frac{x-2}{x-3}$

VA @ $x=3$

$x-3=0$

$x=3$

Just a reminder

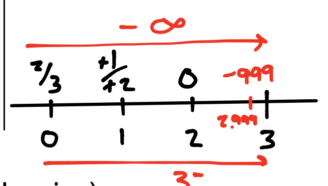
Hole! @ $x=-4$

$x+4=0$

$x=-4$

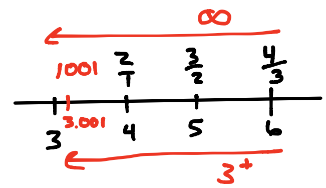
2. Evaluate $\lim_{x\to3^-} f(x) = -\infty$

VA



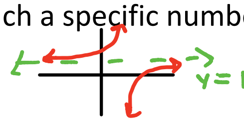
3. Evaluate $\lim_{x\to3^+} f(x) = \infty$

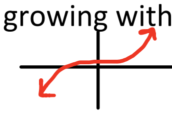
VA



Horizontal Asymptotes: (End-behavior)

What does the y-value approach as the x-value approaches negative infinity AND positive infinity? Does it approach a specific number, or is it growing without bound?





Basic Rules for Horizontal Asymptotes:

Denominator grows faster means $\frac{\text{not as big}}{\text{super duper BIG number!}} = 0$

If the numerator and denominator grow equally fast, then you have $\frac{\text{BIG number!}}{\text{BIG number!}} = 1$

most of time :)

If the numerator grows faster than the denominator, then you have $\frac{\text{BIG number!}}{\text{not as big}} = \infty$

First, you need to recognize which functions grow faster as x-values get larger and larger.

Rank Fastest to Slowest	$f(x)$	$x = 1$	$x = 10$	$x = 100$	$x = 1000$
6	x^2	1	100	10^4	10^6
5	x^3	1	1000	10^6	10^9
4	x^{10}	1	10^{10}	10^{20}	10^{30}
3	2^x	2	1024	$1.3\cdot10^{30}$	Error
2	e^x	2.718	22,026	$2.7\cdot10^{43}$	Error
1	4^x	4	$1.05\cdot10^6$	$1.6\cdot10^{60}$	Error
7	$\ln x$	0	2.303	4.605	6.908

Exponential is fastest

To BIG For CALC

Highlighted largest value in column

Write your questions and thoughts here!

1.3 Asymptotes

Notes

Find the horizontal asymptote(s) of each function.

4. $y = \frac{x^2+4}{3x-5} =$

$x \to +\infty, \quad y = \frac{\text{BIGGER}}{\text{BIG}}$

$x \to -\infty, \quad y = \frac{-\text{BIGGER}}{-\text{BIG}}$

NO HA

5. $y = \frac{x+4}{3x-5}$

$x \to \infty, \quad y = \frac{1 \cdot \text{BIG}}{3 \cdot \text{BIG}}$

$x \to -\infty, \quad y = \frac{1(-\text{BIG})}{3(-\text{BIG})}$

SO HA $y = \frac{1}{3}$

6. $y = \frac{x+4}{3x^2-5} =$

$x \to \infty, \quad y = \frac{\text{BIG}}{\text{BIGGER}}$

$x \to -\infty, \quad y = \frac{-\text{BIG}}{-\text{BIGGER}}$

SO, HA $y =$ ☺

Same question

What about weird ones like this: $y = \frac{\sqrt{x^2}+2}{x-1}$

$x \to \infty, \quad y = \frac{\sqrt{x^2}}{x} = \frac{+x}{+x} = 1$; $x \to -\infty, \quad y = \frac{\sqrt{x^2}}{x} = \frac{+x}{-x} = -1$

SO HA, $y = 1$, $y = -1$

Evaluate the limit.

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7. $\lim_{x\to\infty} \frac{\sqrt{4x^2}+x-2}{3x-1}$

$= \lim_{x\to\infty} \frac{2x}{3x}$

$= \lim_{x\to\infty} \frac{2}{3}$

$= \frac{2}{3}$

8. $\lim_{x\to-\infty} \frac{\sqrt{4x^2}+x-2}{3x-1}$

$= \lim_{x\to-\infty} \frac{2\sqrt{x^2}}{3x}$

$= \frac{2}{3} \cdot \frac{+1}{-1}$

$= -\frac{2}{3}$

9. $\lim_{x\to\infty} -4e^{\frac{1}{x}}$ ($\frac{1}{x} \to 0$)

$= -4e^0$

$= -4 \cdot 1$

$= -4$

10. $\lim_{x\to\infty} 5e^{-x}$

$= \lim_{x\to\infty} \frac{5}{e^x}$

$=$ ☺

Trig Function's Horizontal Asymptotes:

Evaluate the limit.

$-1 \le \sin x \le 1$

11. $\lim_{x\to-\infty} \frac{\sin x}{x} = 0$

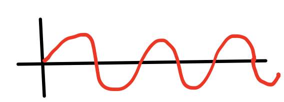
12. $\lim_{x\to\infty} -3\cos\frac{1}{x}$

$= -3 \cdot \cos 0$

$= -3 \cdot 1$

$= -3$

13. $\lim_{x\to\infty} \sin x =$ DNE



14. $\lim_{x\to\infty} 5x \cos x$ ($5x \to \infty$, $\cos x \to$ DNE)

$=$ DNE

Squeeze Theorem: a.k.a. "Sandwich Theorem" or "Pinching Theorem"

If $f(x) \le g(x) \le h(x)$

and if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} h(x) = L$

then $\lim_{x\to a} g(x) = L$

Use the Squeeze Theorem to evaluate the limit.

15. $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right) = 0$

$-1 \le \cos\left(\frac{1}{x^2}\right) \le 1$

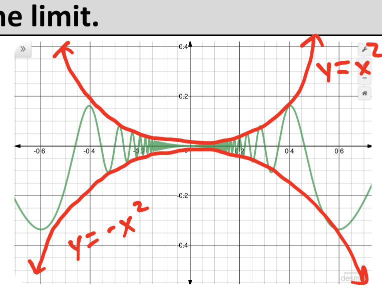
$-x^2 \le x^2\cos\left(\frac{1}{x^2}\right) \le x^2$

$\lim_{x\to 0}(-x^2) \le \lim_{x\to 0} x^2\cos\left(\frac{1}{x^2}\right) \le \lim_{x\to 0} x^2$

$0 = 0 = 0$

FACT

$-1 \le \cos(\text{ANGLE}) \le 1$



Now summarize what you learned!