Calculus

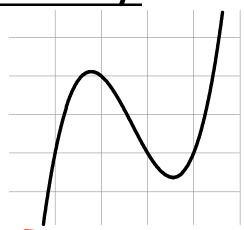
Write your questions and thoughts here!

1.4 Continuity

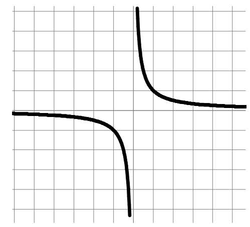
Name:____________________________

Notes

Defining Continuity:



Trace? Yes, so continuous.

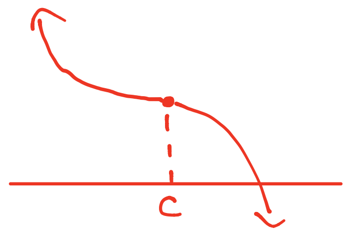


Trace? No, so discontinuous.

Formal Definition of Continuity:

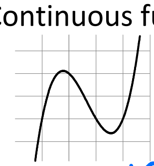
For $f(x)$ to be continuous at $x = c$, the following three conditions must be met:

1. $f(c)$ is defined
2. $\lim_{x\to c} f(x)$ exists
3. $\lim_{x\to c} f(x) = f(c)$

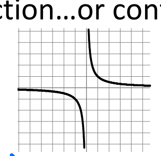


Ask students to name functions

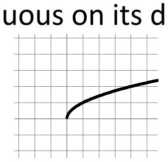
Continuous function…or continuous on its domain?



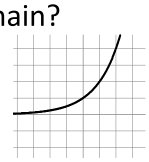
Polynomial (cubic) — Continuous



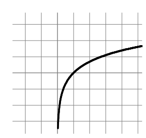
Rational — Discontinuous



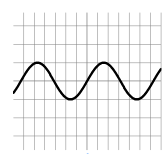
Square root — Continuous on its domain



Exponential — Continuous on its domain



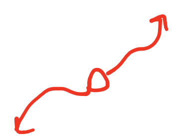
Logarithmic — Continuous on its domain



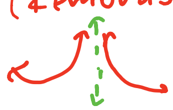
Trig — Continuous

Types of Discontinuities:

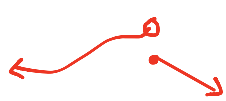
1. Point (HOLE) (Removable)



2. INFINITE (VA)



3. Jump (usually piecewise)



For each function identify the x value and type of each discontinuity.

1. $f(x) = \frac{x^2-8x+12}{x^2+3x-10}$

$= \frac{(x-6)\cancel{(x-2)}}{(x+5)\cancel{(x-2)}}$

$f(x) = \frac{x-6}{x+5}$

VA @ $x=-5$

$x+5=0$

$x=-5$

Hole! $x=2$

$x-2=0$

$x=2$

Infinite Disc @ $x=-5$

Point Disc @ $x=2$

2. $f(x) = \sqrt{2x-3}$

Continuous on its Domain

D: $[\frac{3}{2}, \infty)$

RADICAND ≥ 0

$2x-3 \geq 0$

$2x \geq 3$

$x \geq 3/2$

3. $g(x) = \begin{cases} x^2 - 2x + 1, & x < -1 \text{ (cont.)} \\ x + 2, & -1 \leq x < 2 \text{ (cont.)} \\ 2^x, & x \geq 2 \text{ (cont.)} \end{cases}$

@ $x=-1$

$g(x) = x^2 - 2x + 1$

$g(-1) = (-1)^2 - 2(-1) + 1$

$= 1 + 2 + 1$

$g(-1) = 4$

$g(x) = x + 2$

$g(-1) = -1 + 2$

$g(-1) = 1$

Jump Disc @ $x=-1$

@ $x=2$

$g(x) = x + 2$

$g(2) = (2) + 2$

$g(2) = 4$

$g(x) = 2^x$

$g(2) = 2^{(2)}$

$g(2) = 4$

1.4 Continuity

Notes

Write your questions and thoughts here!

Finding the Domain

Two scenarios to watch for when looking for a **restriction** on the domain.		
1. Denominators	$f(x) = \dfrac{x-5}{x+1}$	Denom $\neq 0$ D: $\mathbb{R}, x \neq -1$
2. RADICALS	$f(x) = \sqrt{7x+3}$	RADICAND ≥ 0 D: $[-3/7, \infty)$

Find the domain of each function.

4. $f(x) = \dfrac{3x}{x\sqrt{x+5}}$

Hole @ $x=0$
$x=0$
$\therefore x \neq 0$

Denom $\neq 0$
$\sqrt{x+5} \neq 0$
$x+5 \neq 0$
$x \neq -5$

RADICAND ≥ 0
$x+5 \geq 0$
$x \geq -5$

D: $x > -5, x \neq 0$

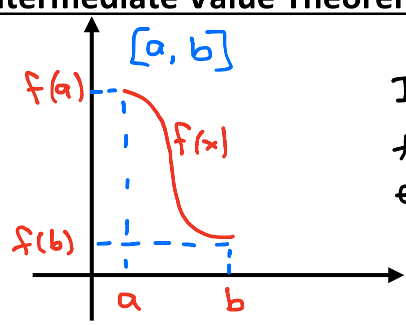
5. $h(x) = \dfrac{5}{2-\sqrt{x}}$

Denom $\neq 0$
$2-\sqrt{x} \neq 0$
$2 \neq \sqrt{x}$
$4 \neq x$

RADICAND ≥ 0
$\sqrt{x} \geq 0$
$x \geq 0$

D: $x \geq 0, x \neq 4$

Intermediate Value Theorem (for continuous functions) - IVT



If f is continuous from a to b, then every y-value between $f(a)$ and $f(b)$ exists at some point in the interval $[a, b]$

6. Use the IVT to answer the following questions if $f(x) = x^3 - 2x - 5$.

a. Find $f(1)$. $= (1)^3 - 2(1) - 5 = 1 - 2 - 5 = -6$

b. Find $f(2)$. $= (2)^3 - 2(2) - 5 = 8 - 4 - 5 = -1$

c. Find $f(3)$. $= (3)^3 - 2(3) - 5 = 27 - 6 - 5 = 16$

d. Does the function have a zero? How do you know?
Does $y=0$

Now summarize what you learned!

Yes. The function is continuous and $f(2) = -1$ and $f(3) = 16$.
The IVT says every y-value must exist between -1 and 16.
Since zero is between -1 and 16, it must exist.