

## 2.1 Average Rate of Change

PRACTICE

### Slope of the Secant Line:

Given a function  $f$ , the equation for the slope of the secant line is

$$\frac{f(x+h) - f(x)}{(x+h) - (x)} = \frac{f(x+h) - f(x)}{h}$$

Now summarize what you learned!

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### 2.1 Average Rate of Change

Calculus

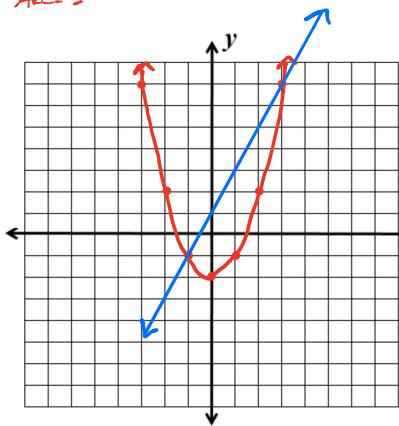
Name: \_\_\_\_\_

Practice

Find the average rate of change for each function on the given interval. On the grid provided, sketch the function and draw the secant line.

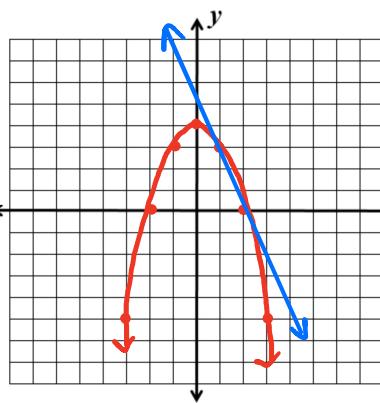
1.  $f(x) = x^2 - 2$ ;  $[-1, 3]$

$$\begin{aligned} ARC &= \frac{f(b) - f(a)}{b - a} \\ &= \frac{[(-1)^2 - 2] - [(-3)^2 - 2]}{(-1) - (-3)} \\ &= \frac{[9 - 2] - [1 - 2]}{4} \\ &= \frac{7 - (-1)}{4} \\ &= \frac{8}{4} \\ ARC &= 2 \end{aligned}$$



2.  $g(x) = 4 - x^2$ ;  $[1, 2]$

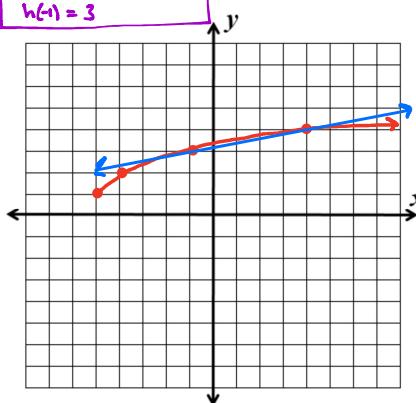
$$\begin{aligned} ARC &= \frac{g(b) - g(a)}{b - a} \\ &= \frac{[4 - (2)^2] - [4 - (1)^2]}{(2) - (1)} \\ &= \frac{[4 - 4] - [4 - 1]}{(2) - (1)} \\ &= \frac{0 - 3}{1} \\ ARC &= -3 \end{aligned}$$



3.  $h(x) = \sqrt{x+5} + 1$ ;  $[-1, 4]$

$$\begin{aligned} h(4) &= \sqrt{4+5} + 1 \\ &= \sqrt{9} + 1 \\ &= 3 + 1 \\ h(4) &= 4 \end{aligned}$$

$$\begin{aligned} ARC &= \frac{h(b) - h(a)}{b - a} \\ &= \frac{h(4) - h(-1)}{(4) - (-1)} \\ &= \frac{(4) - (3)}{5} \\ ARC &= \frac{1}{5} \end{aligned}$$



**Find the average rate of change for each function on the given interval.**

4.  $g(r) = 2r^2 + r - 1$ ;  $[0, 1]$

$$\begin{aligned} g(1) &= 2(1)^2 + (1) - 1 \\ &= 2(1) + 0 \\ g(1) &= 2 \end{aligned}$$
  

$$\begin{aligned} g(0) &= 2(0)^2 + (0) - 1 \\ g(0) &= -1 \end{aligned}$$
  

$$ARC = \frac{g(b) - g(a)}{b-a}$$

$$= \frac{g(1) - g(0)}{(1) - (0)}$$

$$= \frac{(2) - (-1)}{1}$$

$$= \frac{3}{1}$$

$$ARC = 3$$

5.  $s(t) = \frac{1}{t-1}$ ;  $[-5, -2]$

$$\begin{aligned} s(-2) &= \frac{1}{(-2)-1} \\ s(-2) &= -\frac{1}{3} \end{aligned}$$
  

$$\begin{aligned} s(-5) &= \frac{1}{(-5)-1} \\ s(-5) &= -\frac{1}{6} \end{aligned}$$
  

$$ARC = \frac{s(b) - s(a)}{b-a}$$

$$= \frac{s(-2) - s(-5)}{(-2) - (-5)}$$

$$= \frac{(-\frac{1}{3}) - (-\frac{1}{6})}{-3}$$

$$= \frac{-\frac{2}{6} + \frac{1}{6}}{-3}$$

$$= \frac{-\frac{1}{6}}{-3}$$

$$ARC = -\frac{1}{18}$$

6.  $a(x) = \ln x$ ;  $[1, e]$

$$\begin{aligned} a(e) &= \ln(e) \\ a(e) &= 1 \end{aligned}$$
  

$$\begin{aligned} a(1) &= \ln(1) \\ a(1) &= 0 \end{aligned}$$
  

$$ARC = \frac{a(b) - a(a)}{b-a}$$

$$= \frac{a(e) - a(1)}{e-1}$$

$$= \frac{(1) - (0)}{e-1}$$

$$ARC = \frac{1}{e-1}$$

**Find the average rate of change for each function on the given interval. Use appropriate units.**

7.  $s(t) = -t^2 - t + 4$ ;  $[1, 5]$

$t$  represents seconds

$s$  represents feet

$$\begin{aligned} s(5) &= -(5)^2 - (5) + 4 \\ &= -(25) - 1 \\ s(5) &= -26 \end{aligned}$$
  

$$\begin{aligned} s(1) &= -(1)^2 - (1) + 4 \\ &= -(1) + 3 \\ s(1) &= 2 \end{aligned}$$
  

$$ARC = \frac{s(b) - s(a)}{b-a}$$

$$= \frac{s(5) - s(1)}{(5) - (1)}$$

$$= \frac{(-26) - (2)}{4}$$

$$= -\frac{28}{4}$$

$$ARC = -7 \text{ ft/sec}$$

8.  $A(t) = 2^t$ ;  $[2, 4]$

$t$  represents years

$A$  represents dollars

$$\begin{aligned} A(4) &= 2^{(4)} \\ A(4) &= 16 \end{aligned}$$
  

$$\begin{aligned} A(2) &= 2^{(2)} \\ A(2) &= 4 \end{aligned}$$
  

$$ARC = \frac{A(b) - A(a)}{b-a}$$

$$= \frac{A(4) - A(2)}{(4) - (2)}$$

$$= \frac{(16) - (4)}{2}$$

$$= \frac{12}{2}$$

$$ARC = \$6 \text{ per year}$$

9.  $n(m) = \tan m + 4$ ;  $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

$n$  represents nose hairs

$m$  represents months

$$\begin{aligned} n\left(\frac{3\pi}{4}\right) &= \tan\left(\frac{3\pi}{4}\right) + 4 \\ &= -1 + 4 \\ n\left(\frac{3\pi}{4}\right) &= 3 \end{aligned}$$
  

$$\begin{aligned} n\left(\frac{\pi}{4}\right) &= \tan\left(\frac{\pi}{4}\right) + 4 \\ &= 1 + 4 \\ n\left(\frac{\pi}{4}\right) &= 5 \end{aligned}$$
  

$$ARC = \frac{n(b) - n(a)}{b-a}$$

$$= \frac{n\left(\frac{3\pi}{4}\right) - n\left(\frac{\pi}{4}\right)}{\left(\frac{3\pi}{4}\right) - \left(\frac{\pi}{4}\right)}$$

$$= \frac{(3) - (5)}{\frac{2\pi}{4}}$$

$$= -\frac{2}{2} \cdot \frac{2}{\pi}$$

$$ARC = -\frac{2}{\pi} \text{ hairs/month}$$

**Find the equation of the secant line on the given interval. Put the equation in slope-intercept form.**

10.  $v(t) = t^3 - t$ ;  $[-2, 2]$

$$\begin{aligned} v(2) &= (2)^3 - (2) \\ &= 8 - 2 \\ v(2) &= 6 \end{aligned}$$
  

$$\begin{aligned} v(-2) &= (-2)^3 - (-2) \\ &= -8 + 2 \\ v(-2) &= -6 \end{aligned}$$
  

$$ARC = \frac{v(b) - v(a)}{b-a}$$

$$= \frac{v(2) - v(-2)}{(2) - (-2)}$$

$$= \frac{(6) - (-6)}{4}$$

$$= \frac{12}{4}$$

$$ARC = 3$$

11.  $f(x) = \frac{x}{x+2}$ ;  $[-1, 1]$

$$\begin{aligned} f(1) &= \frac{1}{1+2} \\ f(1) &= \frac{1}{3} \end{aligned}$$
  

$$\begin{aligned} f(-1) &= \frac{-1}{(-1)+2} \\ f(-1) &= -\frac{1}{1} \\ f(-1) &= -1 \end{aligned}$$
  

$$ARC = \frac{f(b) - f(a)}{b-a}$$

$$= \frac{f(1) - f(-1)}{(1) - (-1)}$$

$$= \frac{(\frac{1}{3}) - (-1)}{2}$$

$$= \frac{\frac{1}{3} + \frac{3}{3}}{2}$$

$$= \frac{\frac{4}{3}}{2}$$

$$= \frac{4}{6}$$

$$ARC = \frac{2}{3}$$

12.  $h(t) = \sin t$ ;  $\left[\pi, \frac{3\pi}{2}\right]$

$$\begin{aligned} h\left(\frac{3\pi}{2}\right) &= \sin\left(\frac{3\pi}{2}\right) \\ h\left(\frac{3\pi}{2}\right) &= -1 \end{aligned}$$
  

$$\begin{aligned} h(\pi) &= \sin(\pi) \\ h(\pi) &= 0 \end{aligned}$$
  

$$ARC = \frac{h(b) - h(a)}{b-a}$$

$$= \frac{h\left(\frac{3\pi}{2}\right) - h(\pi)}{\left(\frac{3\pi}{2}\right) - (\pi)}$$

$$= \frac{(-1) - (0)}{\frac{3\pi}{2} - \frac{2\pi}{2}}$$

$$= -\frac{1}{\frac{\pi}{2}} \cdot \frac{2}{\pi}$$

$$ARC = -\frac{2}{\pi}$$

Point  $(2, 6)$  or  $(-2, -6)$

Slope  $m = 3$

Point-Slope form  $y - y_1 = m(x - x_1)$

$$y - 6 = 3(x - 2)$$

$$y - 6 = 3x - 6$$

$$y = 3x$$

Point  $(-1, -1)$  or  $(1, \frac{1}{3})$

Slope  $m = \frac{2}{3}$

Point-Slope form  $y - y_1 = m(x - x_1)$

$$y - (-1) = \frac{2}{3}(x - (-1))$$

$$y + 1 = \frac{2}{3}(x + 1)$$

$$y + 1 = \frac{2}{3}x + \frac{2}{3}$$

$$y = \frac{2}{3}x - \frac{1}{3}$$

Point  $(\pi, 0)$  or  $(\frac{3\pi}{2}, -1)$

Slope  $m = -\frac{2}{\pi}$

Point-Slope form  $y - y_1 = m(x - x_1)$

$$y - 0 = -\frac{2}{\pi}(x - \pi)$$

$$y - 0 = -\frac{2}{\pi}(x - \pi)$$

$$y = -\frac{2}{\pi}x + 2$$

Using the interval  $[x, x + h]$ , find the expression that represents the slope of the secant line.

13.  $f(x) = x^2 - x$

$$\begin{aligned} \text{Slope of Secant} &= \frac{f(x+h) - f(x)}{(x+h) - (x)} \\ &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{[(x+h)^2 - (x+h)] - [x^2 - x]}{h} \\ &= \frac{x^2 + 2hx + h^2 - x - h - x^2 + x}{h} \\ &= \frac{2hx + h^2 - h}{h} \\ &= \frac{h(2x + h - 1)}{h} \\ &= 2x + h - 1 \end{aligned}$$

15.  $f(x) = 3 - 2x^2$

$$\begin{aligned} \text{Slope of Secant} &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{[3 - 2(x+h)^2] - [3 - 2x^2]}{h} \\ &= \frac{3 - 2(x^2 + 2hx + h^2) - 3 + 2x^2}{h} \\ &= \frac{3 - 2x^2 - 4hx - 2h^2 - 3 + 2x^2}{h} \\ &= \frac{-4hx - 2h^2}{h} \\ &= \frac{h(-4x - 2h)}{h} \end{aligned}$$

$\text{Slope of Secant} = -4x - 2h$

14.  $f(x) = \sqrt{x}$

$$\begin{aligned} \text{Slope of Secant} &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h} \\ &= \frac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \frac{1}{\sqrt{x+h} + \sqrt{x}} \\ \text{Slope of Secant} &= \frac{1}{\sqrt{x+h} + \sqrt{x}} \end{aligned}$$

16.  $f(x) = \frac{1}{x}$

$$\begin{aligned} \text{Slope of Secant} &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{\left(\frac{1}{x+h} - \frac{1}{x}\right) \cancel{x(x+h)}}{h \cancel{x(x+h)}} \\ &= \frac{x - (x+h)}{h x (x+h)} \\ &= \frac{-h}{h x (x+h)} \\ \text{Slope of Secant} &= \frac{-1}{x(x+h)} \end{aligned}$$

## 2.1 Average Rate of Change

## Test Prep

1. The cost of producing  $x$  units of a certain item is  $c(x) = 2,000 + 8.6x + 0.5x^2$ . What is the average rate of change of  $c$  with respect to  $x$  when the level of production increases from  $x = 300$  to  $x = 310$  units?



(A) 313.6

(B) 310

(C) 214.2

(D) 200

$$ARC = \frac{c(b) - c(a)}{b - a}$$

$$(E) 10 = \frac{c(310) - c(300)}{310 - 300}$$

$$= \frac{52716 - 49580}{10}$$

$$= \frac{3136}{10}$$

$$ARC = 313.6$$