Calculus

2.1 Average Rate of Change

Name:_____

Notes

Recall: Rate of Change

miles | hour

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{dependent unit}}{\text{independent unit}}$$
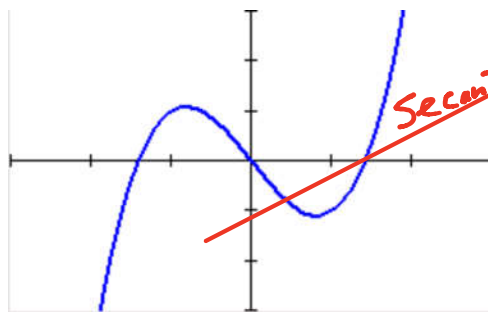
Average Rate of Change:

x-values

A continuous function $f(x)$ on the interval $[a, b]$ has an average rate of change of

Slope between two points

$$\frac{f(b) - f(a)}{b - a} \qquad \text{or} \qquad \frac{f(a) - f(b)}{a - b}$$

This is also the __slope__ of the __secant__ line.

Example 1: Find the average rate of change of $f(x) = x^3 - 2x$ on the interval $\left[\frac{1}{2}, \frac{3}{2}\right]$

Secant

$$ARC = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{f(\frac{3}{2}) - f(\frac{1}{2})}{\frac{3}{2} - \frac{1}{2}}$$

$$= \frac{0.375 - (-0.875)}{\frac{2}{2}}$$

$$f(\tfrac{1}{2}) = -0.875$$

$$= \frac{1.25}{1}$$

$$f(\tfrac{3}{2}) = 0.375$$

$$ARC = 1.25$$

Example 2: The function $h(k) = 3k^2 - k$ represents how long it takes Mr. Brust to clean his house where h is measured in hours and k is measured in the number of kids Mr. Brust has. What is the average rate it takes to clean his house if he has between 1 and 4 kids?

$[1, 4]$

hours | Kids

$$h(4) = 3(4)^2 - (4)$$
$$= 3(16) - 4$$
$$= 48 - 4$$
$$h(4) = 44$$

$$h(1) = 3(1)^2 - (1)$$
$$= 3 \cdot (1) - 1$$
$$= 3 - 1$$
$$h(1) = 2$$

$$ARC = \frac{h(b) - h(a)}{b - a}$$

$$= \frac{h(4) - h(1)}{4 - 1}$$

$$= \frac{44 - 2}{3}$$

$$ARC = \frac{42}{3}$$

$$ARC = 14 \text{ hours}/\text{Kid}$$

2.1 Average Rate of Change

Write your questions
and thoughts here!

Example 3: Find the average rate of change for $f(x) = x^2 - 4x + 1$ on the interval $[x, x+h]$.

$$ARC = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \frac{[(x+h)^2 - 4(x+h) + 1] - [x^2 - 4x + 1]}{h}$$

$$= \frac{x^2 + 2hx + h^2 - 4x - 4h + 1 - x^2 + 4x - 1}{h}$$

$$ARC = \frac{2hx + h^2 - 4h}{h}$$

$$ARC = \frac{2hx + h^2 - 4h}{h}$$

$$= \frac{h(2x + h - 4)}{h}$$

$$ARC = 2x + h - 4$$

Slope of Secant

Slope of the Secant Line:

Given a function f, the equation for the slope of the secant line is

$$\text{Slope of Secant} = \frac{f(x+h) - f(x)}{(x+h) - (x)} = \frac{f(x+h) - f(x)}{h}$$

Now summarize what you learned!

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