2.2 Definition of the Derivative

Calculus Name: ____________________

Practice

Find the derivative using limits. If the equation is given as $y =$, use Leibniz Notation: $\frac{dy}{dx}$. If the equation is given as $f(x) =$, use Lagrange Notation: $f'(x)$. WRITE SMALL!!

1. $f(x) = 7 - 6x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{[7-6(x+h)] - [7-6x]}{h}$$
$$= \lim_{h\to 0} \frac{7-6x-6h-7+6x}{h}$$
$$= \lim_{h\to 0} \frac{-6\cancel{h}}{\cancel{h}}$$
$$= \lim_{h\to 0} -6$$
$$f'(x) = -6$$

2. $y = 5x^2 - x$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{[5(x+h)^2 - (x+h)] - [5x^2 - x]}{h}$$
$$= \lim_{h\to 0} \frac{5(x^2+2hx+h^2) - x - h - 5x^2 + x}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{5x^2} + 10hx + 5h^2 - \cancel{x} - h - \cancel{5x^2} + \cancel{x}}{h}$$
$$= \lim_{h\to 0} \frac{10hx + 5h^2 - h}{h}$$
$$= \lim_{h\to 0} \frac{h(10x + 5h - 1)}{h}$$
$$= \lim_{h\to 0} (10x + 5h - 1)$$
$$= 10x + 5(0) - 1$$
$$\frac{dy}{dx} = 10x - 1$$

3. $y = x^2 + 2x - 9$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{[(x+h)^2 + 2(x+h) - 9] - [x^2 + 2x - 9]}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2hx + h^2 + \cancel{2x} + 2h - \cancel{9} - \cancel{x^2} - \cancel{2x} + \cancel{9}}{h}$$
$$= \lim_{h\to 0} \frac{2hx + h^2 + 2h}{h}$$
$$= \lim_{h\to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$$
$$= \lim_{h\to 0} (2x + h + 2)$$
$$= 2x + (0) + 2$$
$$\frac{dy}{dx} = 2x + 2$$

4. $y = \sqrt{5x + 2}$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\sqrt{5(x+h)+2} - \sqrt{5x+2}}{h} \cdot \frac{(\sqrt{5(x+h)+2} + \sqrt{5x+2})}{(\sqrt{5(x+h)+2} + \sqrt{5x+2})}$$
$$= \lim_{h\to 0} \frac{5x + 5h + 2 - (5x+2)}{h(\sqrt{5(x+h)+2} + \sqrt{5x+2})}$$
$$= \lim_{h\to 0} \frac{\cancel{5x} + 5h + \cancel{2} - \cancel{5x} - \cancel{2}}{h(\sqrt{5(x+h)+2} + \sqrt{5x+2})}$$
$$= \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}(\sqrt{5(x+h)+2} + \sqrt{5x+2})}$$
$$= \lim_{h\to 0} \frac{5}{\sqrt{5(x+h)+2} + \sqrt{5x+2}}$$
$$= \frac{5}{\sqrt{5[x+(0)]+2} + \sqrt{5x+2}}$$
$$= \frac{5}{\sqrt{5x+2} + \sqrt{5x+2}}$$
$$\frac{dy}{dx} = \frac{5}{2\sqrt{5x+2}}$$

5. $f(x) = \frac{1}{x-2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)-2}\cdot\frac{(x-2)}{(x-2)} - \frac{1}{x-2}\cdot\frac{(x+h-2)}{(x+h-2)}}{h} \cdot \frac{(x-2)(x+h-2)}{(x-2)(x+h-2)}$$
$$= \lim_{h\to 0} \frac{(x-2) - (x+h-2)}{h(x-2)(x+h-2)}$$
$$= \lim_{h\to 0} \frac{\cancel{x} - \cancel{2} - \cancel{x} - h + \cancel{2}}{h(x-2)(x+h-2)}$$
$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(x-2)(x+h-2)}$$
$$= \lim_{h\to 0} \frac{-1}{(x-2)(x+h-2)}$$
$$= \frac{-1}{(x-2)(x+(0)-2)}$$
$$= \frac{-1}{(x-2)(x-2)}$$
$$f'(x) = \frac{-1}{(x-2)^2}$$

For each problem, create an equation of the tangent line of f at the given point. Leave in point-slope.

6. $f(7) = 5$ and $f'(7) = -2$

Point: $(7, 5)$

Slope: $m = -2$

Point-Slope form: $y - y_1 = m(x - x_1)$

$y - (5) = -2(x - (7))$

7. $f(-2) = 3$ and $f'(-2) = 4$

Point: $(-2, 3)$

Slope: $m = 4$

Point-Slope form: $y - y_1 = m(x - x_1)$

$y - (3) = 4(x - (-2))$

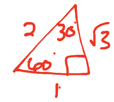
8. $f(x) = 3x^2 + 2x$; $f'(x) = 6x + 2$; $x = -2$

Point:
$$\boxed{\begin{aligned} f(-2) &= 3(-2)^2 + 2(-2) \\ &= 3(4) - 4 \\ &= 12 - 4 \\ f(-2) &= 8 \\ &(-2, 8) \end{aligned}}$$

Slope:
$$\boxed{\begin{aligned} f'(-2) &= 6(-2) + 2 \\ &= -12 + 2 \\ f'(-2) &= -10 \\ m &= -10 \end{aligned}}$$

Point-Slope form: $y - y_1 = m(x - x_1)$

$y - (8) = -10(x - (-2))$



9. $f(x) = 10\sqrt{6x+1}$;
$f'(x) = \frac{30}{\sqrt{6x+1}}$; $x = 4$

Point
$f(4) = 10\sqrt{6(4)+1}$
$= 10\sqrt{24+1}$
$= 10\sqrt{25}$
$= 10(5)$
$f(4) = 50$
$(4, 50)$

Slope
$f'(4) = \frac{30}{\sqrt{6(4)+1}}$
$= \frac{30}{\sqrt{24+1}}$
$= \frac{30}{\sqrt{25}}$
$= \frac{30}{5}$
$f'(4) = 6$
$m = 6$

Point-slope form
$y - y_1 = m(x - x_1)$
$y - (50) = 6(x - (4))$

10. $f(x) = \cos 2x$;
$f'(x) = -2\sin 2x$; $x = \frac{\pi}{4}$

Point
$f(\frac{\pi}{4}) = \cos 2(\frac{\pi}{4})$
$= \cos\frac{\pi}{2}$
$f(\frac{\pi}{4}) = 0$
$(\frac{\pi}{4}, 0)$

Slope
$f'(\frac{\pi}{4}) = -2\sin 2(\frac{\pi}{4})$
$= -2\sin\frac{\pi}{2}$
$= -2(1)$
$f'(\frac{\pi}{4}) = -2$
$m = -2$

Point-slope form
$y - y_1 = m(x - x_1)$
$y - (0) = -2(x - (\frac{\pi}{4}))$

11. $f(x) = \tan x$;
$f'(x) = \sec^2 x$; $x = \frac{\pi}{3}$

Point
$f(\frac{\pi}{3}) = \tan(\frac{\pi}{3})$
$f(\frac{\pi}{3}) = \sqrt{3}$
$(\frac{\pi}{3}, \sqrt{3})$

Slope
$f'(\frac{\pi}{3}) = \sec^2(\frac{\pi}{3})$
$= (2)^2$
$f'(\frac{\pi}{3}) = 4$
$m = 4$

Point-slope form
$y - y_1 = m(x - x_1)$
$y - (\sqrt{3}) = 4(x - (\frac{\pi}{3}))$

Identify the original function $f(x)$, and what value of c to evaluate $f'(c)$.

12.
$$\lim_{h\to 0} \frac{3(1+h)^2 - 7(1+h) + 1 + (3)}{h}$$
$-(-3)$

$f(x) = 3x^2 - 7x + 1$
$f'(1)$

13.
$$\lim_{h\to 0} \frac{\log(2-4(h-5)) - \log(22)}{h}$$

$f(x) = \log(2-4x)$
$f'(-5)$

14.
$$\lim_{x\to -2} \frac{(3x-9x^2)+(42)}{x+2}$$

$f(x) = 3x - 9x^2$
$f(-2)$

15.
$$\lim_{x\to 5} \frac{\frac{1}{\sqrt{3x}} - \frac{1}{\sqrt{15}}}{x-5}$$

$f(x) = \frac{1}{\sqrt{3x}}$
$f'(5)$

16.
$$\lim_{h\to 0} \frac{e^{6(3+h)+1} - e^{19}}{h}$$

$f(x) = e^{6x+1}$
$f'(3)$

17.
$$\lim_{x\to \frac{\pi}{2}} \frac{6x^2\sin x - 3\pi^2}{x - \frac{\pi}{2}}$$

$f(x) = 6x^2 \sin x$
$f'(\frac{\pi}{2})$

For each problem, use the information given to identify the meaning of the two equations in the context of the problem. Write in full sentences!

18. C is the number of championships Sully has won while coaching basketball.
t is the number of years since 2002 for the function $C(t)$.
$C(12) = 3$ and $C'(12) = 0.4$ champs/year
2014 champs 2014

By 2014 Sully has won a total of 3 championships.

In 2014, Sully's total campionships are increasing by 0.4 championships per year.

or

In 2014, Sully's championship rate is 0.4 championships per year.

19. d is the distance (in miles) from home when you walk to school.
h is the number of hours since 7:00 a.m. for the function $d(h)$.
$d(0.2) = 0.5$ and $d'(0.2) = -11$ miles/hour
velocity opposite direction
7:12 Am miles 7:12 Am

At 7:12 Am you are 0.5 miles from home.

At 7:12 Am Your velocity is 11 miles per hour traveling towards home.

or

At 7:12 Am your distance from home is decreasing by 11 miles per hour.

20. W is the number of cartoon shows Mr. Kelly watches every week.
x is the number of children Mr. Kelly has for the function $W(x)$.
$W(7) = 25$ and $W'(7) = 3$ cartoons/week/kid
kids cartoons/week kids

when Kelly has 7 kids, he watches 25 cartoons per week.

when Kelly has 7 kids, his rate of watching cartoons is increasing by 3 cartoons per week per kid.

21. g is the number of gray hairs on Mr. Brust's head.
x is the number of students in his 4th period.
$g(26) = 501$ and $g'(15) = 130$ gray hairs/student
students gray hairs

When Brust has 26 students in 4th period, he has 501 gray hairs.

When Brust has 15 students in 4th period, he is gaining 130 gray hairs per student

2.2 Definition of the Derivative

Test Prep

1. Let $f'(x) = \lim\limits_{h\to 0} \frac{(x+h)^2-x^2}{h}$. For what value of x does $f(x) = 4$?

(A) -4 (B) -1 (C) 1 (D) 2 (E) 4

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2. If $f(x+y) = f(x)\cdot f(y)$ and if $\lim\limits_{h\to 0} \frac{f(h)-1}{h} = 6$, then $f'(x) =$

(A) 6 (B) $6 + f(x)$ (C) $6\cdot f(x)$

(D) $6 + f(h)$ (E) $6\cdot f(h)$

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3. Which of the following gives the derivative of the function $f(x) = x^2$ at the point $(2, 4)$?

(A) $\lim\limits_{h\to 0} \frac{(x+2)^2-x^2}{4}$ (B) $\lim\limits_{h\to \infty} \frac{(2+h)^2-2^2}{h}$ (C) $\frac{(2+h)^2-2^2}{h}$

(D) $\lim\limits_{h\to 0} \frac{(2+h)^2-2^2}{h}$ (E) $\lim\limits_{h\to 0} \frac{(4+h)^2-4^2}{h}$