Calculus

Write your questions and thoughts here!

2.2 Definition of the Derivative

Name:________________________

Notes

Recall: Average rate of change = Slope of Secant line

Average rate of change on the interval [x , x+h] is $\frac{f(x+h)-f(x)}{\underbrace{(x+h)-(x)}_{\text{cancel}}} = \frac{y_2-y_1}{x_2-x_1} = \frac{\Delta y}{\Delta x}$

Defintion of the Derivative:

This limit gives an expression that calculates the instantaneous rate of change (slope of the tangent line) of $f(x)$ at any given x-value.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Notation for the Derivative:

Δ = delta

Lagrange	Leibniz
$f'(x)$ "f prime of x" y' "y prime"	$\frac{dy}{dx}$ "dee y over dee x" "derivative of y with respect to x"

Find the derivative using the Definition of the Derivative (limits).

1. $f(x) = 2x^2 - 7x + 1$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{[2(x+h)^2 - 7(x+h) + 1] - [2x^2 - 7x + 1]}{h}$

$= \lim_{h\to 0} \frac{2(x^2+2hx+h^2) - 7x - 7h + 1 - 2x^2 + 7x - 1}{h}$

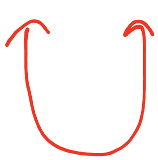
$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4hx + 2h^2 - \cancel{7x} - 7h + \cancel{1} - \cancel{2x^2} + \cancel{7x} - \cancel{1}}{h}$

$= \lim_{h\to 0} \frac{h(4x+2h-7)}{h}$

$= \lim_{h\to 0} (4x+2h-7)$

$= 4x + 2(0) - 7$

$f'(x) = 4x - 7$



2. $y = \frac{1}{x}$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \quad x(x+h) \quad x(x+h)$

$= \lim_{h\to 0} \frac{x-(x+h)}{hx(x+h)}$

$= \lim_{h\to 0} \frac{\cancel{x} - \cancel{x} - h}{hx(x+h)}$

$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}x(x+h)}$

$= \lim_{h\to 0} \frac{-1}{x(x+h)}$

$= \frac{-1}{x[x+(0)]}$

$\frac{dy}{dx} = \frac{-1}{x^2}$

3. If f represents how many meters you have run and x represents the minutes, describe in full sentences the following:

f = meters ran
x = minutes ran $f(8) = 1{,}500$

At 8 min you've run 1500 meters.

f = meters ran $\frac{df}{dx}$
x = minutes ran

$f'(3) = 161$ meters/min

At 3 minutes, you're running at 161 m/min.

At 3 minutes, the total meters run is increasing by 161 meters each minute.

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Alternate Definition – Derivative at a Point:

Finding the derivative at a specific x-value ($x = c$).

$$f'(c) = \lim_{h\to 0}\frac{f(c+h)-f(c)}{h} \quad \text{or} \quad f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

$y_2 - y_1$

$x_2 - x_1$

4. Find $f'(-2)$ if $f(x) = 2x^2 + 1$.

$f'(c) = \lim_{x\to c}\frac{f(x)-f(c)}{x-c}$

$= \lim_{x\to -2}\frac{(2x^2+1)-[2(-2)^2+1]}{x-(-2)}$

$= \lim_{x\to -2}\frac{(2x^2+1)-9}{x+2}$

$= \lim_{x\to -2}\frac{2x^2-8}{x+2}$

$= \lim_{x\to -2}\frac{2(x^2-4)}{x+2}$

$= \lim_{x\to -2}\frac{2(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

$= \lim_{x\to -2} 2(x-2)$

$= 2[(-2)-2]$

$\to f'(-2) = 2(-4)$

$f'(-2) = -8$

Side work

$2(-2)^2+1$

$= 2(4)+1$

$= 8+1$

$= 9$

5. $f(x) = x^3 - \frac{3}{x}$ and $f'(x) = 3x^2 + \frac{3}{x^2}$

Find the equation of the tangent line at $x = 2$.

Point $(2, \frac{13}{2})$

$f(2) = (2)^3 - \frac{3}{2}$

$f(2) = 8 - \frac{3}{2}$

$f(2) = \frac{16}{2} - \frac{3}{2}$

$f(2) = \frac{13}{2}$

Slope $m = \frac{51}{4}$

$f'(2) = 3(2)^2 + \frac{3}{(2)^2}$

$= 3(4) + \frac{3}{4}$

$= 12 + \frac{3}{4}$

$= \frac{48}{4} + \frac{3}{4}$

$f'(2) = \frac{51}{4}$

Point-slope form

$y - y_1 = m(x - x_1)$

$y - (\frac{13}{2}) = \frac{51}{4}(x - (2))$

Identify the original function $f(x)$, and what value of c to evaluate $f'(c)$.

6. $\lim_{h\to 0}\frac{3\ln(2+h) - 3\ln 2}{h}$

$f(x) = 3\ln x$

$f'(2)$

7. $\lim_{x\to 7}\frac{\frac{1}{\sqrt{x^2-2x}} - \frac{1}{\sqrt{35}}}{x-7}$

$f(x) = \frac{1}{\sqrt{x^2-2x}}$

$f'(7)$

Now summarize what you learned!