

2.3 Differentiability

Calculus

Name: _____

Practice

Given $f(x)$ and $f'(x)$ on a given interval $[a, b]$, find a value c that satisfies the Mean Value Theorem.

$$1. f(x) = -x^2 + 4x - 2; [-1, 2]$$

$$f'(x) = -2x + 4$$

$$\begin{aligned} f(2) &= -(2)^2 + 4(2) - 2 \\ &= -4 + 8 - 2 \\ f(2) &= 2 \end{aligned}$$

$$\begin{aligned} f(-1) &= -(-1)^2 + 4(-1) - 2 \\ &= -1 - 4 - 2 \\ f(-1) &= -7 \end{aligned}$$

$$\begin{aligned} ARC &= \frac{f(b) - f(a)}{b - a} \\ &= \frac{f(2) - f(-1)}{(2) - (-1)} \\ &= \frac{(2) - (-7)}{3} \\ &= \frac{9}{3} \\ ARC &= 3 \end{aligned}$$

MVT

$$\begin{aligned} ARC &= f'(c) \\ 3 &= -2c + 4 \\ -1 &= -2c \\ \frac{1}{2} &= c \end{aligned}$$

$$2. f(x) = \frac{x^2}{2} + 4x + 7; [-7, -3]$$

$$f'(x) = x + 4$$

$$\begin{aligned} f(-3) &= \frac{(-3)^2}{2} + 4(-3) + 7 \\ &= \frac{9}{2} - 12 + 7 \\ &= \frac{9}{2} - \frac{10}{2} \\ f(-3) &= \frac{-1}{2} \end{aligned}$$

$$\begin{aligned} f(-7) &= \frac{(-7)^2}{2} + 4(-7) + 7 \\ &= \frac{49}{2} - 28 + 7 \\ &= \frac{49}{2} - \frac{42}{2} \\ f(-7) &= \frac{7}{2} \end{aligned}$$

$$\begin{aligned} ARC &= \frac{f(b) - f(a)}{b - a} \\ &= \frac{f(-3) - f(-7)}{(-3) - (-7)} \\ &= \frac{\left(\frac{-1}{2}\right) - \left(\frac{7}{2}\right)}{4} \\ &= \frac{-8}{4} \\ &= -2 \\ ARC &= -2 \end{aligned}$$

MVT

$$\begin{aligned} ARC &= f'(c) \\ -1 &= c + 4 \\ -5 &= c \end{aligned}$$

$$3. f(x) = -2x^2 + 12x - 15; [2, 4]$$

$$f'(x) = -4x + 12$$

$$\begin{aligned} f(4) &= -2(4)^2 + 12(4) - 15 \\ &= -32 + 48 - 15 \\ &= -30 + 33 \\ f(4) &= 1 \end{aligned}$$

$$\begin{aligned} f(2) &= -2(2)^2 + 12(2) - 15 \\ &= -8 + 24 - 15 \\ &= -8 + 9 \\ f(2) &= 1 \end{aligned}$$

$$\begin{aligned} ARC &= \frac{f(b) - f(a)}{b - a} \\ &= \frac{f(4) - f(2)}{(4) - (2)} \\ &= \frac{(1) - (1)}{2} \\ &= \frac{0}{2} \\ ARC &= 0 \end{aligned}$$

MVT

$$\begin{aligned} ARC &= f'(c) \\ 0 &= -4c + 12 \\ 4c &= 12 \\ c &= 3 \end{aligned}$$

$$4. f(x) = x^3 - 12x; [-2, 2]$$

$$f'(x) = 3x^2 - 12$$

MVT

$$\begin{aligned} f(2) &= (2)^3 - 12(2) \\ &= 8 - 24 \\ f(2) &= -16 \\ f(-2) &= (-2)^3 - 12(-2) \\ &= -8 + 24 \\ f(-2) &= 16 \\ ARC &= \frac{f(b) - f(a)}{b - a} \\ &= \frac{(-16) - (16)}{(2) - (-2)} \\ &= \frac{-32}{4} \\ ARC &= -8 \end{aligned}$$

$$5. f(x) = \frac{1}{x}; [-2, 2]$$

$$f'(x) = -\frac{1}{x^2}$$

Not a continuous function.
Therefore MVT does not apply.

$$6. f(x) = x^3 + 24x - 16; [0, 4]$$

$$f'(x) = 3x^2 + 24$$

$$\begin{aligned} f(4) &= (4)^3 + 24(4) - 16 \\ &= 64 + 96 - 16 \\ f(4) &= 144 \end{aligned}$$

$$\begin{aligned} f(0) &= (0)^3 + 24(0) - 16 \\ f(0) &= -16 \end{aligned}$$

$$\begin{aligned} ARC &= \frac{f(b) - f(a)}{b - a} \\ &= \frac{f(4) - f(0)}{4 - 0} \\ &= \frac{(144) - (-16)}{4} \\ &= \frac{160}{4} \\ ARC &= 40 \end{aligned}$$

MVT

$$\begin{aligned} ARC &= f'(c) \\ 40 &= 3c^2 + 24 \\ 16 &= 3c^2 \\ \frac{16}{3} &= c^2 \\ \pm \sqrt{\frac{16}{3}} &= c \\ \pm \frac{4}{\sqrt{3}} &= c \\ \text{not } c \in [0, 4] \\ c &= \frac{4}{\sqrt{3}} \end{aligned}$$

Using a calculator find the value of the derivative at a given point. DON'T show any work. You should be able to quickly find the answer with a calculator.

$$7. f(x) = x^2 + 5x$$

$$f'(1.98) = 8.96$$

$$8. f(x) = \csc 5x$$

$$f'\left(\frac{\pi}{2}\right) = 0$$

$$9. f(x) = \ln x$$

$$f'(205) = 0.005$$

$$10. f(x) = \frac{1}{x}$$

$$f'(\sqrt{2}) = -0.5$$

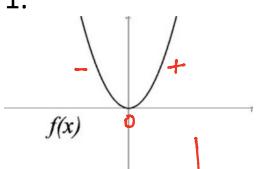
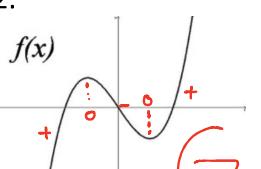
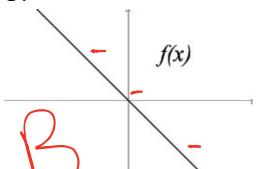
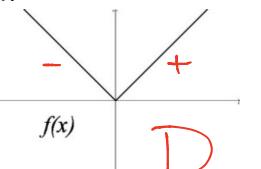
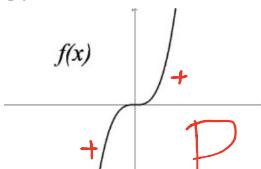
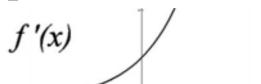
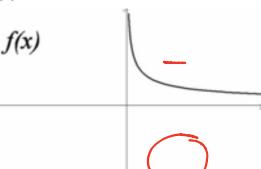
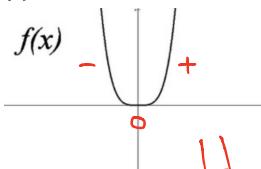
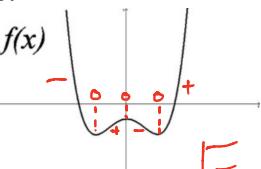
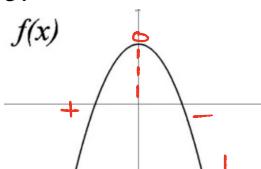
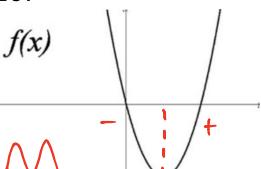
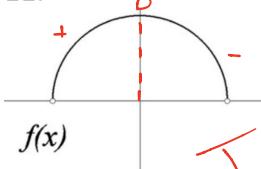
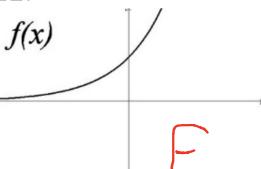
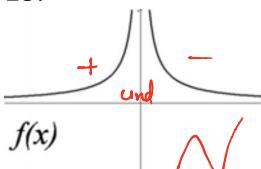
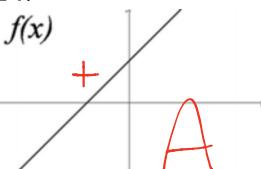
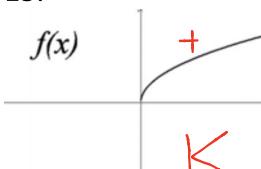
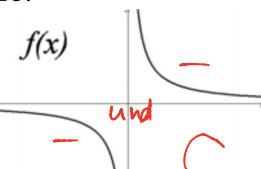
$$11. f(x) = e^{7x}$$

$$f'(1.5) = 254,210.595$$

$$12. f(x) = 8x^2 - 5x^3$$

$$f'\left(\frac{1}{3}\right) = 3.667$$

Match each function with the graph of its derivative.

Function		Derivative	
1.		1.	A 
2.		2.	B  3
3.		3.	C  15
4.		4.	D  4
5.		5.	E 
6.		6.	F  8
7.		7.	G  2
8.		8.	H  7
9.		9.	I 
10.		10.	J  11
11.		11.	K  15
12.		12.	L  9
13.		13.	M 
14.		14.	N  10
15.		15.	O  6
16.		16.	P  5