3.1 Power Rule

PRACTICE

Find the derivative of the following.

1. $f(x) = 2x^3 - 4x + 5$

$f'(x) = 6x - 4$

2. $y = 3x^{100} - 2x^8 - 7x$

$\frac{dy}{dx} = 300x^{99} - 16x^7 - 7$

3. $g(x) = 5x^{-2} - \frac{1}{2}x^4$

$g'(x) = -10x^{-3} - 2x^3$

4. $h(x) = \frac{x^6}{3} + 6x^{2/3} - 4x^{1/2} + 2$

$h(x) = \frac{1}{3}x^6 + 6x^{2/3} - 4x^{1/2} + 2$

$h'(x) = 2x^5 + 4x^{-1/3} - 2x^{-1/2}$

5. $f(x) = \frac{1}{x^3} + \frac{12}{x}$

$f(x) = x^{-3} + 12x^{-1}$

$f'(x) = -3x^{-4} - 12x^{-2}$

$f'(x) = \frac{-3}{x^4} - \frac{12}{x^2}$

6. $y = \frac{3}{x^{-2}} - \frac{1}{(6x)^2}$

$y = 3x^2 - \frac{1}{36}x^{-2}$

$\frac{dy}{dx} = 6x + \frac{1}{18}x^{-3}$

$\frac{dy}{dx} = 6x + \frac{1}{18x^3}$

7. $f(x) = \sqrt{x} + 3\sqrt[3]{x} + 2$

$f(x) = x^{1/2} + 3x^{1/3} + 2$

$f'(x) = \frac{1}{2}x^{-1/2} + 1 \cdot x^{-2/3}$

$f'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}$

8. $y = \sqrt[3]{x^2} + 8\sqrt[4]{x^7}$

$y = x^{2/3} + 8x^{7/4}$

$\frac{dy}{dx} = \frac{2}{3}x^{-1/3} + 14x^{3/4}$

$\frac{dy}{dx} = \frac{2}{3\sqrt[3]{x}} + 14\sqrt[4]{x^3}$

9. $f(x) = \frac{1}{\sqrt{x}} + \frac{3}{6x}$

$f(x) = x^{-1/2} + \frac{1}{2}x^{-1}$

$f'(x) = -\frac{1}{2}x^{-3/2} - \frac{1}{2}x^{-2}$

$f'(x) = \frac{-1}{2\sqrt{x^3}} - \frac{1}{2x^2}$

10. $f(x) = \frac{1}{\sqrt{x}} + \frac{3}{\sqrt[5]{x^2}}$

$f(x) = x^{-1/2} + 3x^{-2/5}$

$f'(x) = -\frac{1}{2}x^{-3/2} - \frac{6}{5}x^{-7/5}$

$f'(x) = \frac{1}{2\sqrt{x^3}} - \frac{6}{5\sqrt[5]{x^7}}$

11. $s(t) = -16t^2 + 40t + 5$

$s'(t) = -32t + 40$

12. $y = \pi x^2 - \pi$

Coefficient Constant

$\frac{dy}{dx} = 2\pi x$

13. $V(r) = \frac{4}{3}\pi r^3$

Coefficient variable

$V'(r) = 4\pi r^2$

14. $f(x) = \frac{2x^3 + 4x - 5}{x}$

$f(x) = \frac{2x^3}{x} + \frac{4x}{x} - \frac{5}{x}$

$f(x) = 2x^2 + 4 - 5x^{-1}$

$f'(x) = 4x + 5x^{-2}$

$f'(x) = 4x + \frac{5}{x^2}$

15. $g(x) = \frac{6x^3 + 4x^2 - 9x}{3}$

$g(x) = \frac{6x^3}{3} + \frac{4x^2}{3} - \frac{9x}{3}$

$g(x) = 2x^3 + \frac{4}{3}x^2 - 3x$

$g'(x) = 6x^2 + \frac{8}{3}x - 3$

Find the derivatives of the following.

16. $f(x) = 3x^7 - 4x^3 + 5x + 7$

$f'(x) = 21x^6 - 12x^2 + 5$

$f''(x) = 126x^5 - 24x$

$f'''(x) = 630x^4 - 24$

$f^{(4)}(x) = 2520x^3$

17. $y = 4\sqrt{x} + e$

$y = 4x^{1/2} + e$

$\frac{dy}{dx} = 2x^{-1/2} = \frac{2}{\sqrt{x}}$

$\frac{d^2y}{dx^2} = -x^{-3/2} = \frac{-1}{\sqrt{x^3}}$

18. $y = \frac{1}{x^3} - \frac{1}{2}x^4 + ex^2$

$y = x^{-3} - \frac{1}{2}x^4 + ex^2$

$y' = -3x^{-4} - 2x^3 + 2ex$

$y' = \frac{-3}{x^4} - 2x^3 + 2ex$

$y'' = 12x^{-5} - 6x^2 + 2e$

$y'' = \frac{12}{x^5} - 6x^2 + 2e$

$y''' = -60x^{-6} - 12x$

$y''' = \frac{-60}{x^6} - 12x$

Given $f(x) = 3x^2 - x + 2$, $g(x) = \frac{1}{x^3} + e^2$, and $h(x) = \sqrt{x}$, find the following.

19. $f'(2) = 11$

$f'(x) = 6x - 1$

$f'(2) = 6(2) - 1$
$= 12 - 1$
$f'(2) = 11$

20. $g'''(-3) = \frac{-60}{729}$

$g(x) = x^{-3} + e^2$
$g' = -3x^{-4}$
$g'' = 12x^{-5}$
$g''' = -60x^{-6}$

$g'''(-3) = \frac{-60}{(-3)^6}$
$= \frac{-60}{729}$

21. $2h''(4) = \frac{-1}{16}$

$h(x) = x^{\frac{1}{2}}$
$h' = \frac{1}{2}x^{-\frac{1}{2}}$
$h'' = -\frac{1}{4}x^{-\frac{3}{2}}$
$h'' = \frac{-1}{4\sqrt{x^3}}$

$2h''(4) = 2 \cdot \left(\frac{-1}{4(\sqrt{4})^3}\right)$
$= \frac{-1}{2(2)^3}$
$= \frac{-1}{2 \cdot 8}$
$2h''(4) = -\frac{1}{16}$

22. Find the slope of $f(x)$ at $x = 3$.

$f'(3) = 6(3) - 1$
$= 18 - 1$
$f'(3) = 17$

Slope of $f(x)$ at $x = 3$ is 17.

23. At what value of x is $f'(x) = 0$?

$0 = 6x - 1$
$1 = 6x$
$\frac{1}{6} = x$

24. What is the slope of the tangent line of $h(x)$ at the point (16, 4) ?

Slope
$h' = \frac{1}{2\sqrt{x}}$
$h'(16) = \frac{1}{2\sqrt{16}}$
$h'(16) = \frac{1}{2 \cdot 4}$
$h'(16) = \frac{1}{8}$

The slope of the tangent line of $h(x)$ at $(16, 4)$ is $\frac{1}{8}$.

Find the slope of the tangent line.

25. $f(x) = 2\sqrt{x} - \pi^2$

$f(x) = 2x^{\frac{1}{2}} - \pi^2$
$f'(x) = x^{-\frac{1}{2}}$
$f'(x) = \frac{1}{\sqrt{x}}$

26. $y = -2x^3 + \frac{1}{2}x^2 - 7x + 5$

$\frac{dy}{dx} = -6x^2 + x - 7$

27. $g(x) = \frac{1}{x^2} - \frac{1}{2x}$

$g(x) = x^{-2} - \frac{1}{2}x^{-1}$
$g'(x) = -2x^{-3} + \frac{1}{2}x^{-2}$
$g'(x) = \frac{-2}{x^3} + \frac{1}{2x^2}$

Is the slope of the tangent line positive, negative, or zero at the given point?

28. $f(x) = \frac{4x^3 - 16x^2}{2x}$ at $x = 2$

$f(x) = \frac{4x^3}{2x} - \frac{16x^2}{2x}$
$f(x) = 2x^2 - 8x$
$f'(x) = 4x - 8$

Sign Analysis
$f'(x) = 4(x-2)$
$f'(2) = +(0) = 0$

The slope is zero.

29. $y = 2x^4 + 5x^3$ at $x = -2$

$y' = 8x^3 + 15x^2$

Sign Analysis
$y' = x^2(8x + 15)$
$y'(-2) = (+)(-)$
$y'(-2) = -$

The slope is negative.

30. $g(x) = 3\sqrt[3]{x^5} - 4x^{-1}$ at $x = 8$

$g(x) = 3x^{5/3} - 4x^{-1}$
$g'(x) = 5x^{2/3} + 4x^{-2}$

Sign Analysis
$g'(x) = x^{-2}(5x^{8/3} + 4)$
$g'(x) = \frac{\frac{5}{(\sqrt[3]{x})^4} + 4}{x^2}$
$g'(8) = \frac{+}{+}$
$g'(8) = +$

The slope is positive

Write the equation of the tangent line and the normal line at the point given.

31. $f(x) = 3\sqrt{x} + 4$ at $x = 4$

$f(x) = 3x^{\frac{1}{2}} + 4$

Point
$f(4) = 3\sqrt{4} + 4$
$= 3(2) + 4$
$= 6 + 4$
$f(4) = 10$
$(4, 10)$

Slope
$f'(x) = \frac{3}{2}x^{-\frac{1}{2}}$
$f'(x) = \frac{3}{2\sqrt{x}}$
$f'(4) = \frac{3}{2\sqrt{4}}$
$= \frac{3}{2(2)}$
$f'(4) = \frac{3}{4}$
$m = \frac{3}{4}$
$\perp m = -\frac{4}{3}$

Tangent
$y - y_1 = m(x - x_1)$
$y - 10 = \frac{3}{4}(x - 4)$

Normal
$y - y_1 = m(x - x_1)$
$y - 10 = \frac{-4}{3}(x - 4)$

32. $y = \frac{x^2 + 3x - 4}{2}$ at $x = 8$

$y = \frac{1}{2}x^2 + \frac{3}{2}x - 2$

Point
$y(8) = \frac{1}{2}(8)^2 + \frac{3}{2}(8) - 2$
$= \frac{1}{2}(64) + 12 - 2$
$= 32 + 10$
$y(8) = 42$
$(8, 42)$

Slope
$\frac{dy}{dx} = x + \frac{3}{2}$
$\left.\frac{dy}{dx}\right|_{x=8} = (8) + \frac{3}{2}$
$= \frac{16}{2} + \frac{3}{2}$
$\left.\frac{dy}{dx}\right|_{x=8} = \frac{19}{2}$
$m = \frac{19}{2}$
$\perp m = \frac{-2}{19}$

tangent
$y - y_1 = m(x - x_1)$
$y - 42 = \frac{19}{2}(x - 8)$

Normal
$y - y_1 = m(x - x_1)$
$y - 42 = \frac{-2}{19}(x - 8)$

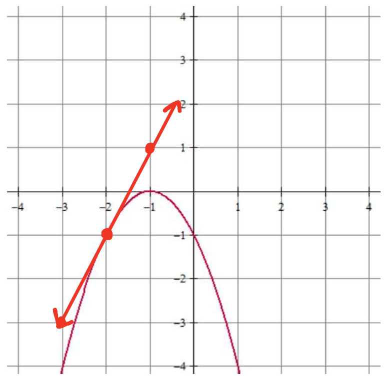
The function is graphed below. Write the equation of the tangent line at the given point and graph it.

33. $f(x) = -x^2 - 2x - 1$ at $x = -2$

Point
$(-2,-1)$
use graph

Slope
$f' = -2x - 2$
$f'(-2) = -2(-2) - 2$
$= 4 - 2$
$f'(-2) = 2$
$m = 2$

tangent
$y - y_1 = m(x - x_1)$
$y - (-1) = 2(x - (-2))$
$y + 1 = 2x + 4$
$y = 2x + 3$

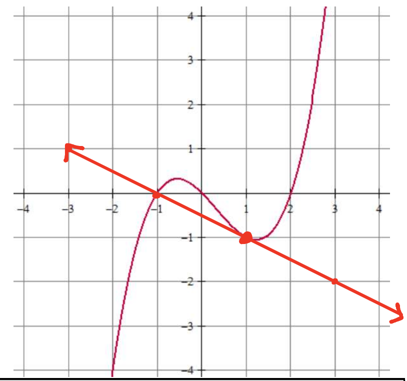


34. $y = \frac{x^3}{2} - \frac{x^2}{2} - x$ at $x = 1$

Point
$(1,-1)$

Slope
$\frac{dy}{dx} = \frac{3}{2}x^2 - x - 1$
$\left.\frac{dy}{dx}\right|_{x=1} = \frac{3}{2}(1)^2 - (1) - 1$
$= \frac{3}{2}(1) - 2$
$= \frac{3}{2} - \frac{4}{2}$
$\left.\frac{dy}{dx}\right|_{x=1} = -\frac{1}{2}$

tangent
$y - (-1) = -\frac{1}{2}(x - 1)$
$y + 1 = -\frac{1}{2}x + \frac{1}{2}$
$y = -\frac{1}{2}x - \frac{1}{2}$



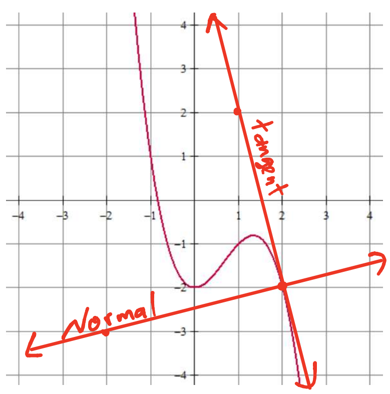
The function is graphed below. Write the equation of the normal line at the given point and graph it.

35. $f(x) = -x^3 + 2x^2 - 2$ at $x = 2$

Point
$(2,-2)$

Slope
$f' = -3x^2 + 4x$
$f'(2) = -3(2)^2 + 4(2)$
$= -3(4) + 8$
$= -12 + 8$
$f'(2) = -4$
$\perp m = \frac{1}{4}$

Normal
$y - y_1 = m(x - x_1)$
$y - (-2) = \frac{1}{4}(x - 2)$
$4y + 8 = x - 2$
$4y = x - 10$
$y = \frac{1}{4}x - \frac{5}{2}$

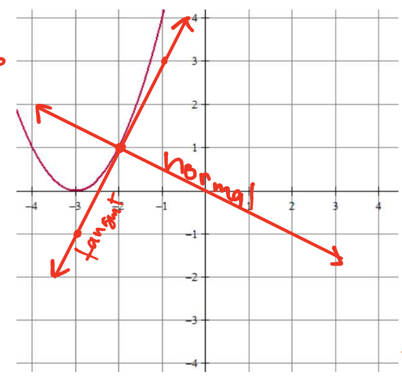


36. $y = x^2 + 6x + 9$ at $x = -2$

Point
$(-2,1)$

Slope
$y' = 2x + 6$
$y'(-2) = 2(-2) + 6$
$= -4 + 6$
$y'(-2) = 2$
$\perp m = -\frac{1}{2}$

Normal
$y - y_1 = m(x - x_1)$
$y - 1 = -\frac{1}{2}(x - (-2))$
$y - 1 = -\frac{1}{2}x - 1$
$y = -\frac{1}{2}x$



You will need to use a graphing calculator for 37-42

Use the graph to find the derivative of the function at the given value. Round to nearest thousandth.

37. $f(x) = \frac{x^2+1}{x-2}$ at $x = 6$

$f'(6) = 0.688$

38. $y = e^x$ at $x = -1$

$\left.\frac{dy}{dx}\right|_{x=-1} = 0.368$

39. $f(\theta) = 2\sin\theta$ at $\theta = \frac{\pi}{2}$

RAD!

$f'(\frac{\pi}{2}) = 0$

Write the equation of the tangent line at the point given and sketch the graph. Round to nearest thousandth.

40. $f(x) = -\sqrt{3x+4}$ at $x = 5$

$f(5) = -4.359$
$f'(5) = -0.344$

$y - y_1 = m(x - x_1)$
$y - (-4.359) = -0.344(x - 5)$
or
$y = -0.344x - 2.639$

41. $y = \ln(x) + 4$ at $x = e$

$y(e) = 5$
$y'(e) = 0.368$

$y - y_1 = m(x - x_1)$
$y - 5 = 0.368(x - e)$
or
$y = 0.368x + 4$

42. $f(\theta) = \csc\theta + 1$ at $\theta = \frac{\pi}{4}$

$f(\frac{\pi}{4}) = 2.414$
$f'(\frac{\pi}{4}) = -1.414$

$y - y_1 = m(x - x_1)$
$y - 2.414 = -1.414(x - \frac{\pi}{4})$
or
$y = -1.414x + 3.525$

MULTIPLE CHOICE

1. Let $f(x) = x^3 + 2x - 5$. What is the x-coordinate of a point where the instantaneous rate of change of f is the same as the average rate of change of f on the interval $-1 < x < 1$?

   (A) $\frac{\sqrt{3}}{3}$
   (B) $\frac{1}{2}$
   (C) 0
   (D) $\frac{1}{3}$
   (E) $\sqrt{3}$

Mean Value Theorem

Average Rate of Change $= f'(c)$

$3 = 3c^2 + 2$

$1 = 3c^2$

$\frac{1}{3} = c^2$

$\pm\sqrt{\frac{1}{3}} = c$, Not a choice, so rationalize $c = \frac{\pm 1}{\sqrt{3}} = \frac{\pm\sqrt{3}}{3}$

Instantaneous Rate Change

$f'(x) = 3x^2 + 2$

Average Rate Change

$f(x) = x^3 + 2x - 5$

$f(1) = (1)^3 + 2(1) - 5$
$= 1 + 2 - 5$
$f(1) = -2$

$f(-1) = (-1)^3 + 2(-1) - 5$
$= -1 - 2 - 5$
$f(-1) = -8$

$ARC = \frac{\Delta y}{\Delta x}$
$= \frac{(-2) - (-8)}{(1) - (-1)}$
$= \frac{6}{2}$
$ARC = 3$

2. Given $g(x) = 2x^5 + \frac{b}{x^2}$ where b is a constant, find the value of b if $g'(2) = 180$.

   (A) 10
   (B) 20
   (C) -40
   (D) -80
   (E) none of the above

1) Find g'
2) Substitute $g'(2) = 180$ into g'
3) Solve for b.

$g(x) = 2x^5 + bx^{-2}$

$g'(x) = 10x^4 - 2bx^{-3}$

$180 = 10(2)^4 - \frac{2b}{(2)^3}$

$180 = 10(16) - \frac{2b}{8}$

$180 = 160 - \frac{b}{4}$

$20 = -\frac{b}{4}$

$-80 = b$

3. Given $f'(x) = \frac{1}{x}$ and $f(2) = 5$, write an equation for the line which is tangent to the graph of $f(x)$ at the point where $x = 2$.

   (A) $y = \frac{1}{2}x - \frac{1}{2}$
   (B) $y = \frac{1}{5}x + 5$
   (C) $y = \frac{1}{2}x + 4$
   (D) $y = \frac{1}{5}x - \frac{23}{5}$
   (E) $y = \frac{1}{2}x + 5$

1) Find Point
2) Find Slope
3) Use Point-Slope Form → Slope-int

Point

$(2, 5)$

Slope

$f'(x) = \frac{1}{x}$

$f'(2) = \frac{1}{2}$

Point-Slope

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{2}(x - 2)$

$y - 5 = \frac{1}{2}x - 1$

$y = \frac{1}{2}x + 4$

4. If the line normal to the graph of f at the point $(1,2)$ passes through the point $(-1,1)$, then which of the following gives the value of $f'(1) = ?$

   (A) -2
   (B) 2
   (C) $-\frac{1}{2}$
   (D) $\frac{1}{2}$
   (E) 3

1) Find slope of Normal

2) Find ⊥ tangent slope

normal Slope $= \frac{\Delta y}{\Delta x}$

$= \frac{(2) - (1)}{(1) - (-1)}$

$= \frac{1}{2}$

⊥ tangent $m = -2$

 **You are allowed to use a graphing calculator for #5** 

5. Which of the following is an equation of the line tangent to the graph of $f(x) = x^6 - x^4$ at the point where $f'(x) = -1$?

(A) $y = -x - 1.031$

(B) $y = -x - 0.836$

(C) $y = -x + 0.836$

(D) $y = -x + 0.934$

(E) $y = -x + 1.031$

1) FIND POINT
2) FIND Slope
3) Point-slope → slope-int

Point

$f'(x) = 6x^5 - 4x^3$

$-1 = 6x^5 - 4x^3$

CALC

$(-.934, -.097)$

$f(-.934) = -.097$

Do not write on AP Test

$Y_2 = 6x^5 - 4x^3$

$Y_3 = -1$

$\boxed{2nd}$ $\boxed{CALC}$ intersect

m of tangent

$m = -1$

tangent line

$y - y_1 = m(x - x_1)$

$y - (-.097) = -1(x - (-.934))$

$y + .097 = -x - .934$

$y = -x - 1.031$

FREE RESPONSE

Your score: ____ out of 5

Use the table to answer the questions below.

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$
0	-10	1	2	-7	3	-4
2	-4	5	2	-1	7	8
5	20	11	2	83	58	26

1. Find the average rate of change of f over the interval $0 \le x \le 2$. Find the value of x at which the instantaneous velocity of g is equal to the average rate of change of f over the interval $0 \le x \le 2$.

$ARC = \frac{\Delta y}{\Delta x}$

$= \frac{(-10)-(-4)}{(0)-(2)}$

$= \frac{-6}{-2}$

$ARC = 3$

x	$f(x)$
0	-10
2	-4

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$
0	-10	1	2	-7	3	-4
2	-4	5	2	-1	7	8
5	20	11	2	83	58	26

$x = 0$

2. Write the equation of the line normal to $g(x)$ at the point where $x = 2$.

Point

$g(2) = -1$

Slope of tangent

$g'(2) = 7$

Normal slope

$m = -\frac{1}{7}$

Normal Line

$y - y_1 = m(x - x_1)$

$y - (-1) = -\frac{1}{7}(x - 2)$

$y + 1 = -\frac{1}{7}(x - 2)$

$7y + 7 = -x + 2$

$7y = -x - 5$

$y = -\frac{1}{7}x - \frac{5}{7}$

3. $(f + g)''(5) = f''(5) + g''(5)$

$= 2 + 26$

$= 28$

4. If $B = f(x) - 2g(x)$, then $B'(0) = -5$

$B' = f'(x) - 2g'(x)$

$B'(0) = f'(0) - 2g'(0)$

$= 1 - 2 \cdot 3$

$= 1 - 6$

$B'(0) = -5$