3.1 Power Rule

NOTES

CALCULUS

Write your questions here!

Notation

Euler $D_{x,y}$ $\dot{f}$ Newt

f'	$f'(x)$	y'	$\frac{dy}{dx}$
f prime	f prime of x	y prime	$d\left(\frac{y}{dx}\right)$

POWER RULE

$$x^n = n \cdot x^{n-1}$$

Find the derivative of the following.

$f(x) = 3x^7 - 4x^5 - \frac{1}{3}x^3 + x^2 - 3x + 7$

$f'(x) = 21x^6 - 20x^4 - x^2 + 2x - 3$

$y = 2x^{-3} + 4x + \pi$

$\frac{dy}{dx} = -6x^{-4} + 4$

Rewrite and then take the derivative.

$y = \sqrt[3]{x^7} - \sqrt{x} + 2\sqrt[5]{x^2}$

$y = x^{7/3} - x^{1/2} + 2x^{2/5}$

FR $\frac{dy}{dx} = \frac{7}{3}x^{4/3} - \frac{1}{2}x^{-1/2} + \frac{4}{5}x^{-3/5}$

$\frac{dy}{dx} = \frac{7}{3}\sqrt[3]{x^4} - \frac{1}{2\sqrt{x}} + \frac{4}{5\sqrt[5]{x^3}}$

$g(x) = \frac{1}{x} + \frac{4}{x^2} - \frac{1}{(3x)^2}$

$g(x) = x^{-1} + 4x^{-2} - \frac{1}{9}x^{-2}$

$\frac{dg}{dx} = -x^{-2} - 8x^{-3} + \frac{2}{9}x^{-3}$

$\frac{dg}{dx} = -\frac{1}{x^2} - \frac{8}{x^3} + \frac{2}{9x^3}$

$f(x) = \frac{-16x^2+5x-1}{2x}$

$f(x) = -8x + \frac{5}{2} - \frac{1}{2}x^{-1}$

$\frac{df}{dx} = -8 + \frac{1}{2}x^{-2}$

$\frac{df}{dx} = -8 + \frac{1}{2x^2}$

Evaluate

$f(x) = \frac{1}{2}x^4 - 4x^{-2} + e$

Find $f'(3)$ $f'(x) = 2x^3 + 8x^{-3}$

$f'(3) = 2(3)^3 + \frac{8}{(3)^3}$

$f'(3) = 2(27) + \frac{8}{27}$

$f'(3) = 54 + \frac{8}{27}$

$y = \frac{1}{\sqrt{x}} + 4x$

Find $\left.\frac{dy}{dx}\right|_{x=4}$

$y = x^{-1/2} + 4x$

$\frac{dy}{dx} = -\frac{1}{2}x^{-3/2} + 4$

$\frac{dy}{dx} = -\frac{1}{2\sqrt{x^3}} + 4$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{-1}{2(\sqrt{4})^3} + 4$

$= \frac{-1}{2(2)^3} + 4$

$= \frac{-1}{2(8)} + 4$

$= -\frac{1}{16} + \frac{64}{16}$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{63}{64}$

Higher Order Derivatives

$f(x) = x^7 - 2x^4 + 5x^2 - 3x + 9$

$f'(x) = 7x^6 - 8x^3 + 10x - 3$

$f''(x) = 42x^5 - 24x^2 + 10$

$f'''(x) = 210x^4 - 48x$

$f^{(4)}(x) = 840x^3 - 48$

$y = \sqrt{x} + x^{-2} = x^{1/2} + x^{-2}$

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - 2x^{-3} = \frac{1}{2\sqrt{x}} - \frac{2}{x^3}$

$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-3/2} + 6x^{-4} = \frac{-1}{4\sqrt{x^3}} + \frac{6}{x^4}$

Find Derivative on the Calculator

$f(x) = \frac{1}{2}x\sqrt{2x-1}$

$f'(4) =$ 2.079

$f'(e) =$ 1.698

$f(\theta) = 1 + \csc\theta$

$f'(\pi) =$ DNE

$f'\left(\frac{\pi}{2}\right) =$ 0

Derivative means…

Slope at a point

Given $y = \frac{1}{2}x^4 - x + 2$ find the slope at $x = 2$

$y' = 2x^3 - 1$

$y'(2) = 2(2)^3 - 1$
$= 2(8) - 1$
$= 16 - 1$
$y'(2) = 15$

The slope is 15 at $x = 2$

Slope of the tangent line

Write the equation of the line tangent to $y = \frac{1}{2}x^4 - x + 2$ at $x = 2$

Point
$y(2) = \frac{1}{2}(2)^4 - (2) + 2$
$= \frac{1}{2}(16) + 0$
$y(2) = 8$
$(2, 8)$

Slope
$m = 15$

Point-slope form
$y - y_1 = m(x - x_1)$
$y - (8) = 15(x - 2)$

Instantaneous rate of change

What is the instantaneous rate of change at 3 seconds?
$s(t) = -4.9t^2 + 40t + 6$

$s'(t) = -9.8t + 40$
$s'(3) = -9.8(3) + 40$
$= -29.4 + 40$
$s'(3) = 10.6$ units/second

Normal Line

⊥ to tangent line =

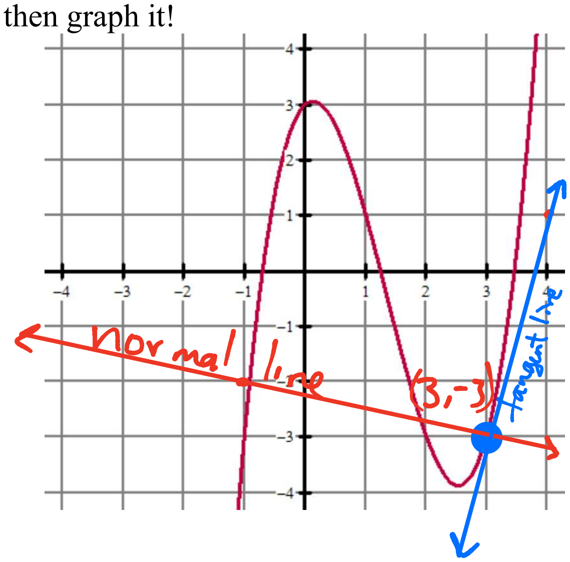
Write the equation of the normal line at $x = 3$ and then graph it!

$f(x) = x^3 - 4x^2 + x + 3$

Point
$f(3) = (3)^3 - 4(3)^2 + (3) + 3$
$= 27 - 4(9) + 6$
$= 33 - 36$
$f(3) = -3$ $(3, -3)$

Slope
$f'(x) = 3x^2 - 8x + 1$
$f'(3) = 3(3)^2 - 8(3) + 1$
$= 3(9) - 24 + 1$
$= 27 - 23$
$f'(3) = 4$
$\perp m = -\frac{1}{4}$

Point-slope
$y - y_1 = m(x - x_1)$
$y - (-3) = -\frac{1}{4}(x - (3))$
$y + 3 = -\frac{1}{4}(x - 3)$



Derivative Rules

Constant Rule $\frac{d}{dx}c = 0$

Constant Multiple Rule $\frac{d}{dx}(cu) = c\frac{du}{dx}$

Power Rule $\frac{d}{dx}x^n = nx^{n-1}$

Sum/Difference Rule $\frac{d}{dx}(u \pm v) = \left(\frac{du}{dx} \pm \frac{dv}{dx}\right)$

SUMMARY:

Now, summarize your notes here!