3.2 Product and Quotient Rule

CALCULUS

Write your questions here!

Find the derivative.

$$f(x) = (x+4)(2x-5)$$

Using power rule

$f(x) = x^2 + 3x - 20$

$f'(x) = 2x + 3$

PRODUCT RULE

$$\frac{d}{dx}(uv) = \frac{d}{dx}u \cdot v + u \cdot \frac{d}{dx}v$$

or

$$= u' \cdot v + u \cdot v'$$

Find the derivative of the following.

$$f(x) = (3x^2 + 2x - 3)(x-1)$$

$f'(x) = (3x^2+2x-3)'(x-1) + (3x^2+2x-3)(x-1)'$

$= (6x+2)(x-1) + (3x^2+2x-3)(1)$

$= 6x^2 + 4x - 2 + 3x^2 + 2x - 3$

$f'(x) = 9x^2 + 6x - 5$

$$y = (2x^{-3} + 4x + \pi)(4x+1)$$

$\frac{dy}{dx} = \frac{d}{dx}(2x^{-3}+4x+\pi)\cdot(4x+1) + (2x^{-3}+4x+\pi)\cdot\frac{d}{dx}(4x+1)$

$= (-6x^{-4}+4)(4x+1) + (2x^{-3}+4x)(4)$

$= -24x^{-3} - 6x^{-4} + 16x + 4 + 8x^{-3} + 16x$

$\frac{dy}{dx} = -6x^{-4} - 16x^{-3} + 32x + 4$

Evaluate

$$f(x) = \sqrt{x}(3x^2 - 3)$$

Find $f'(4)$

$f'(x) = (x^{\frac{1}{2}})'(3x^2-3) + (x^{\frac{1}{2}})(3x^2-3)'$

$= \frac{1}{2}x^{-\frac{1}{2}}(3x^2-3) + x^{\frac{1}{2}}(6x)$

$= \frac{3}{2}x^{-1} - \frac{3}{2}x^{-\frac{1}{2}} + 6x^{\frac{3}{2}}$

$f'(x) = \frac{3}{2x} - \frac{3}{2\sqrt{x}} + 6\sqrt{x^3}$

$f'(4) = \frac{3}{2(4)} - \frac{3}{2\sqrt{4}} + 6(\sqrt{4})^3$

$= \frac{3}{8} - \frac{3}{2\cdot 2} + 6(2)^3$

$= \frac{3}{8} - \frac{3}{4} + 6(8)$

$= \frac{3}{8} - \frac{6}{8} + \frac{384}{8}$

$f'(4) = \frac{381}{8}$

Find the derivative.

$$f(x) = \frac{x-5}{2x+1}$$

$f'(x) = \frac{(x-5)'(2x+1) - (x-5)(2x+1)'}{(2x+1)^2}$

$= \frac{(1)(2x+1) - (x-5)(2)}{(2x+1)^2}$

$= \frac{2x+1-2x+10}{(2x+1)^2}$

$f'(x) = \frac{11}{(2x+1)^2}$

QUOTIENT RULE

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{\frac{d}{dx}u \cdot v - u \cdot \frac{d}{dx}v}{v^2}$$

or

$$= \frac{u' \cdot v - u \cdot v'}{v^2}$$

Find the derivative of the following.

$$f(x) = \frac{3x+1}{2x^2}$$

$$f(x) = \frac{3x}{2x^2} + \frac{1}{2x^2}$$

$$f(x) = \frac{3}{2}x^{-1} + \frac{1}{2}x^{-2}$$

$$f'(x) = -\frac{3}{2}x^{-2} - x^{-3}$$

$$f'(x) = \frac{-3}{2x^2} - \frac{1}{x^3}$$

$$y = \frac{2x^2}{3x+1}$$

$$\frac{dy}{dx} = \frac{\frac{d}{dx}(2x^2)\cdot(3x+1) - 2x^2\frac{d}{dx}(3x+1)}{(3x+1)^2}$$

$$= \frac{4x(3x+1) - 2x^2(3)}{(3x+1)^2}$$

$$= \frac{12x^2+4x-6x^2}{(3x+1)^2}$$

$$\frac{dy}{dx} = \frac{6x^2+4x}{(3x+1)^2}$$

Horizontal Tangents

Find all horizontal tangents for $y = \frac{2x^2}{3x+1}$

$$\frac{dy}{dx} = \frac{(2x^2)'(3x+1) - 2x(3x+1)'}{(3x+1)^2}$$

$$\frac{dy}{dx} = \frac{4x(3x+1) - 2x(3)}{(3x+1)^2}$$

$$\frac{dy}{dx} = \frac{12x^2+4x-6x}{(3x+1)^2}$$

$$\frac{dy}{dx} = \frac{12x^2-2x}{(3x+1)^2}$$

A horizontal tangent has a slope of zero.

$$\frac{dy}{dx} = \frac{12x^2-2x}{(3x+1)^2}$$

$$0 = \frac{2x(6x-1)}{(3x+1)^2}$$

ZON

$2x = 0$, $x = 0$; $6x-1=0$, $6x = 1$, $x = \frac{1}{6}$

ZOD

Later!

$\therefore$ Horizontal tangents @ $x = 0, \frac{1}{6}$

Find $f'(4)$ given the following:

$$g(4) = 3 \text{ and } g'(4) = -2$$

$$h(4) = -1 \text{ and } h'(4) = 5$$

$$f(x) = g(x) - h(x)$$

$$f'(x) = g'(x) - h'(x)$$

$$f'(4) = g'(4) - h'(4)$$

$$= (-2) - (5)$$

$$f'(4) = -7$$

$$f(x) = h(x) + 2$$

$$f(4) = h(4) + 2$$

$$= (-1) + 2$$

$$f(4) = 1$$

$$f(x) = g(x) + 2h(x)$$

$$f(4) = g(4) + 2h(4)$$

$$= (3) + 2(-1)$$

$$= 3 - 2$$

$$f(4) = 1$$

$$f(x) = \frac{h(x)}{g(x)}$$

$$f'(x) = \frac{h'(x)g(x) - h(x)g'(x)}{g^2(x)}$$

$$f'(4) = \frac{h'(4)g(4) - h(4)g'(4)}{g^2(4)}$$

$$= \frac{(5)(3) - (-1)(-2)}{(3)^2}$$

$$= \frac{15-2}{9}$$

$$f'(4) = \frac{13}{9}$$

$$f(x) = g(x)h(x)$$

$$f'(x) = h'(x)g(x) + h(x)g'(x)$$

$$f'(4) = h'(4)g(4) + h(4)g'(4)$$

$$= (5)(3) + (-1)(-2)$$

$$= 15 + 2$$

$$f'(4) = 17$$

SUMMARY:

Now, summarize your notes here!