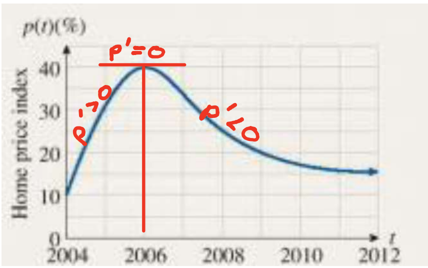
Use the information given to answer the following.

1. The home price index as a percentage change from 2003 in year t, is represented by $y = p(t)$.

$p = \%$

$t = \text{year}$



a) What year does $p'(t) = 0$? 2006

b) Is $p'(2008)$ positive, negative, or zero?

c) Find the average rate of change from 2006 to 2008.

$ARC = \frac{p(b)-p(a)}{b-a} = \frac{p(2008)-p(2006)}{(2008)-(2006)} = \frac{(25)-(40)}{2} = \frac{-15\%}{2\text{ year}}$

2. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = -t^3 + 7t^2 - 16t + 8$ where s is measured in meters and t is measured in seconds.

$s = \text{meters}$
$t = \text{seconds}$

a) Find the instantaneous velocity at any time t.

$v(t) = -3t^2 + 14t - 16$

b) Find the acceleration of the particle at any time t.

$a(t) = -6t + 14$

c) When is the particle at rest? (FIND $V = 0$)

$0 = -3t^2 + 14t - 16$

$0 = (-3t^2 + 8t) + (6t - 16)$

$0 = -t(3t-8) + 2(3t-8)$

$0 = (3t-8)(-t+2)$

$0 = 3t - 8$; $-t + 2 = 0$

$8 = 3t$; $2 = t$

$8/3 = t$

At 2 & 8/3 seconds

d) What is the displacement of the particle for the first 3 seconds?

$S(3) = -(3)^3 + 7(3)^2 - 16(3) + 8$

$= -(27) + 7(9) - 48 + 8$

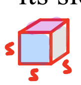
$= -27 + 63 - 40$

$S(3) = -4$

3. Write the surface area, A, of a cube as function of its side, s, measured in inches.

$A = in^2$

$s = in$



$A(s) = 6 \cdot s^2$

a) Find the instantaneous rate of change of surface area with respect to its side when $s = 4$ in.

$\left.\frac{dA}{ds}\right|_{s=4} = 12s\Big|_{s=4} = 48\ in^2$ per in

b) Evaluate the rate of change of A at $s = 1$ and $s = 5$.

$\left.\frac{dA}{ds}\right|_{s=1} = 12(1) = 12\ in^2$ per in

$\left.\frac{dA}{ds}\right|_{s=5} = 12(5) = 60\ in^2$ per in

4. The position, in meters, of a body at time t sec is $s(t) = t^3 - 6t^2 + 9t$. Find the body's acceleration each time the velocity is zero.

$\boxed{V = 3t^2 - 12t + 9 \quad a = 6t - 12}$

$s = \text{meters}$

$t = \text{seconds}$

when is $v = 0$?

$0 = 3t^2 - 12t + 9$

$0 = (3t^2 - 3t) + (-9t + 9)$

$0 = 3t(t-1) + -9(t-1)$

$0 = (t-1)(3t-9)$

$0 = t - 1$; $0 = 3(t-3)$

$1 = t$; $0 = t - 3$; $3 = t$

$\boxed{a(1) = 6(1) - 12 = 6 - 12 \quad a(1) = -6\ m/s^2}$

$\boxed{a(3) = 6(3) - 12 = 18 - 12 \quad a(3) = 6\ m/s^2}$

5. The following table shows oil production by Pemex, Mexico's national oil company, for 2001-2007 ($t = 1$ represents 2001)

	2001	2003	2005	2007
***t* (year since 2000)**	1	3	5	7
***P* (million gallons/day)**	3.1	3.4	3.5	3.7

a) Approximate $P'(2)$. Label and justify!

$P'(2) \approx ARC(1,3) = \frac{P(b)-P(a)}{b-a}$

$= \frac{P(3)-P(1)}{(3)-(1)}$

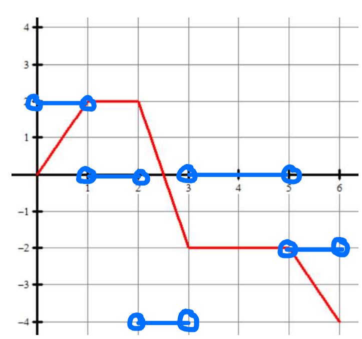
$= \frac{(3.4)-(3.1)}{2}$

$= \frac{.3}{2}$

$= .15$ million gallons/day/year

The oil production is growing by about 150,000 gallons per day each year

6. A particle P moves on the number line. The graph $s = f(t)$ shows the position of P as function of time t.



a) When is P moving to the left? (Negative slope)

$(2,3) \cup (5,6)$

b) When is P moving to the right? (Positive slope)

$(0,1)$

c) When is P standing still? (slope = 0)

$(1,2) \cup (3,5)$

d) Graph the particle's velocity where defined.

7. The number of iPods sold by Apple each year from 2004 through 2007 can be approximated by $f(t) = -t^2 + 20t + 3$ in millions of iPods where $t = 0$ represents 2004. f = million iPods

t = time after 2004 $f' > 0$

a) Is the number of iPods sold in 2006 increasing or decreasing? $f' < 0$

$f'(t) = -2t + 20$

$f'(2) = -2(2) + 20$
$= -4 + 20$
$f'(2) = 16$

increasing

b) What is the average rate of change from 2004-2007?

$\boxed{f(3) = -(3)^2 + 20(3) + 3 = -(9) + 60 + 3, \quad f(3) = 54}$

$\boxed{f(0) = -(0)^2 + 20(0) + 3, \quad f(0) = 3}$

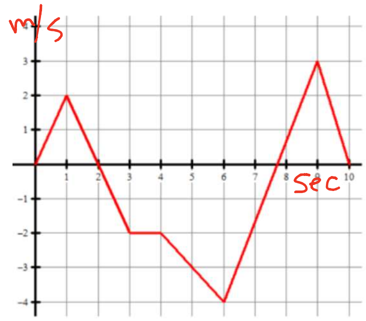
$ARC = \frac{f(b) - f(a)}{b - a}$
$= \frac{f(3) - f(0)}{(3) - (0)}$
$= \frac{(54) - (3)}{3}$
$= \frac{51}{3}$

ARC = 17 million iPods/year

8. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.

f = meters
t = seconds

a) When does the body reverse direction? At 2 and 7.6 seconds



b) When is the body moving at a constant speed?

(3, 4)

c) What is the body's maximum speed?

4 m/s

d) What time interval(s) is the body speeding up?

$(0,1) \cup (2,3) \cup (4,6) \cup (7.6, 9)$

9. A rock thrown vertically upward from the surface of the moon at a velocity of 32 meters per second reaches a height of $s(t) = 32t - 0.8t^2$ meters in t seconds.

S = meters
t = seconds

a) Find the rock's velocity and acceleration as functions of time. $V = 32 - 1.6t$

$a = -1.6$

b) How long did it take the rock to reach its highest point? (Highest point is a max. A max has horizontal tangent line.)

$v(t) = 32 - 1.6t$
$0 = 32 - 1.6t$
$1.6t = 32$
$t = 20$ seconds

10. The table shows the stopping distance d, in meters, of an automobile traveling at a speed v, in kilometers per hour.

d = meters
v = km/h

velocity, v	20	40	60	80	100
distance, d	2.3	8.9	20.2	35.9	56.7

a) Approximate $d'(50)$. Label and justify!

$d'(50) \simeq ARC(40, 60) = \frac{d(60) - d(40)}{(60) - (40)}$

$= \frac{(20.2) - (8.9)}{20}$

$d'(50) \simeq \frac{11.3 \text{ meters}}{20 \text{ Km/hr}}$

11. The data in the table gives selected values for the velocity, in meters per minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t. V = m/min

t = min

Time t	0	2	5	6	8	12
Velocity $v(t)$	-3	2	3	5	7	5

a) At $t = 0$, is the particle moving to the right or left? Explain. At $v(0) = -3$. Since the velocity is negative, the particle is moving left.

b) Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Justify.

Yes, between $t=0$ and $t=2$ the velocity changes from negative to positive. Since the function is differentiable and continuous, the I.V.T. applies. The IVT states that every velocity between -3 and 2 must exist on the time interval $(0,2)$. Since 0 is between -3 and 2, the particle must be at rest on the time interval $(0,2)$

c) Use the data from the data to approximate $v'(10)$. Explain the meaning of $v'(10)$ in terms of the particle motion.

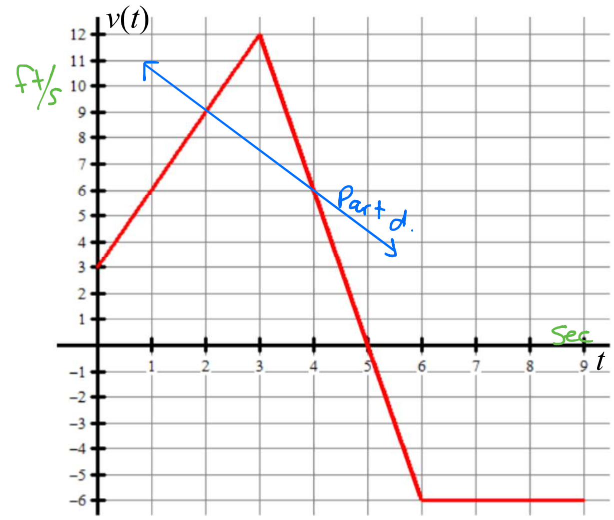
$v'(10) = ARC(8,12) = \frac{v(12) - v(8)}{(12) - (8)} = \frac{(5) - (7)}{4} = \frac{-2}{4} = \frac{-1}{2} \frac{\text{m/min}}{\text{min}}$

At 10 minutes, the velocity is decreasing by $\frac{1}{2}$ m/min every second. (or 1 m/s every 2 seconds)

d) Let $a(t)$ denote the acceleration of the particle at time t. Is there guaranteed to be a time $t = c$ in the interval $0 \le t \le 12$ such that $a(c) = 0$? Justify. Yes.

The function is differentiable, so the MVT applies. The A.R.C. on $[6,12]$ is 0, because $\frac{v(12) - v(6)}{12 - 6} = \frac{5-5}{6} = 0$. The MVT thus says that there exist a point c such that $v'(c) = 0$.

12. The graph represents the velocity, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ to $t = 9$ seconds.



a) At $t = 4$, is the particle moving to the right or left? Explain.

At $t=4$, the particle is moving to the right because it has a positive velocity.

b) Over what time interval is the particle moving left? Explain.

The particle is moving left from 5 seconds to 9 seconds because it has a negative velocity.

c) At $t = 4$, is the acceleration positive or negative? Explain.

The accelaration is negative because the slope of the velocity is a negative number

d) What is the average acceleration of the particle over the interval $2 \le t \le 4$? Show the computations and label your answer. Acceleration is the rate of change of velocity.

$$ARC = \frac{v(4)-v(2)}{(4)-(2)} = \frac{6-9}{2} = \frac{-3}{2}\frac{\text{ft/sec}}{\text{sec}}$$

e) Is there guaranteed to be a time t in the interval $2 \le t \le 4$ such that $v'(t) = -\frac{3}{2}$ ft/sec^2? Justify.

NO, the velocity is not differentiable at $t=3$ due to a corner point. Thus the IVT does not apply.

f) At what time t in the given interval is the particle farthest to the right. Explain.

The particle is farthest to the right at 5 seconds. That is when it stops going right and goes left.

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13. A particle moves along the x-axis so that at time t its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$ where t is measured in seconds and x is measured in feet

$x(t) = ft$ $\quad$ $x =$ feet

$t = sec$ $\quad$ $t =$ seconds

a) At $t = 0$, is the particle moving to the right or left? Explain.

$v(t) = 3t^2 - 12t + 9$

$\boxed{v(0) = 3(0)^2 - 12(0) + 9 \quad v(0) = 9}$

The particle is moving right because the velocity is positive

b) At $t = 1$, is the velocity of the particle increasing or decreasing? Explain.

$a(t) = 6t - 12$ $\boxed{a(1) = 6(1) - 12 = 6 - 12 \quad a(1) = -6}$

The velocity is decreasing at 1 second because the acceleration is negative.

c) Find all values of t for which the particle is moving left.

If moving left, $v<0$.

$3t^2 - 12t + 9 < 0$

$3(t^2 - 4t + 3) < 0$

$3(t-3)(t-1) < 0$

Boundary

$t-3=0 \quad t-1=0$

$t=3 \quad t=1$

$3(t-3)(t-1) < 0$

$(+)(-)(-) = +$ | $(+)(-)(+) = -$ | $(+)(+)(+) = +$

0 1 2 3 4

$1 < t < 3$

d) What is the displacement of the first 6 seconds?

End − beginning

Displacement $= x(6) - x(0)$

$= 65\text{ ft} - 11\text{ ft}$

$= 54\text{ ft}$

$x(6) = (6)^3 - 6(6)^2 + 9(6) + 11$

$= 0 + 54 + 11$

$x(6) = 65\text{ ft}$

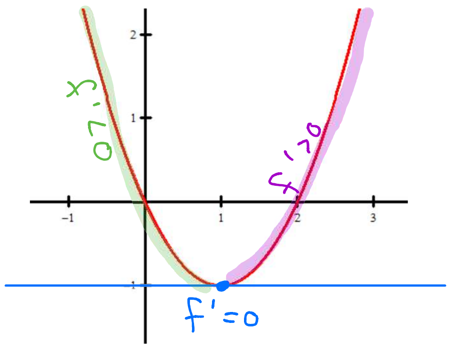
$x(0) = (0)^3 - 6(0)^2 + 9(0) + 11$

$x(0) = 11\text{ ft}$

MULTIPLE CHOICE

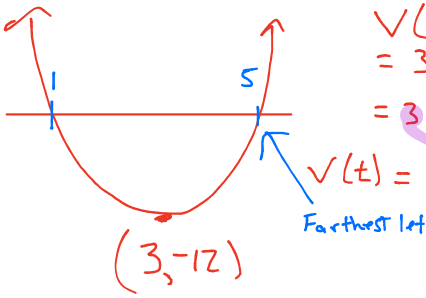
1. The graph of the differentiable function $y = f(x)$ is shown below. Which of the following is true?

(A) $f'(0) > f(0)$ (neg > 0) F
(B) $f'(1) < f(1)$ (0 < -1) F
(C) $f'(2) < f(2)$ (pos < 0) F
(D) $f'(1) = f(0)$ (0 = 0) T
(E) $f'(2) = f(2)$ (pos = 0) F



2. The position of the particle traveling along a straight line is $x(t) = t^3 - 9t^2 + 15t + 3$. On the interval $t = 0$ to $t = 10$, when is the particle farthest to the left?

(A) $t = 0$
(B) $t = 1$
(C) $t = 3$
(D) $t = 5$
(E) $t = 10$



$V(t) = 3t^2 - 18t + 15$
$= 3(t^2 - 6t \quad) + 15$
$= 3[t^2 - 6t + 9] + 15 - 27$
$V(t) = 3[t-3]^2 - 12$

$0 = 3(t-3)^2 - 12$
$12 = 3(t-3)^2$
$4 = (t-3)^2$
$\pm 2 = t - 3$
$+2 = t - 3$ or $-2 = t - 3$
$5 = t$ $\quad 1 = t$

3. If the position of an ant traveling along a horizontal path at time t is $3t^2 + 1$, what is the ant's average velocity from $t = 1$ to $t = 6$?

(A) $\frac{1}{21}$
(B) 6
(C) $\frac{109}{6}$
(D) 21
(E) 220

$AV = ARC = \dfrac{[3(6)^2+1] - [3(1)^2+1]}{(6)-(1)}$
$= \dfrac{[3(36)+1] - [3(1)+1]}{5}$
$= \dfrac{[108+1] - [3+1]}{5}$
$= \dfrac{109 - 4}{5}$
$= \dfrac{105}{5}$
$AV = 21$

4. Find all values of c that satisfy the Mean Value Theorem for $f(x) = x^3 + 1$ on $[-2, 4]$.

(A) $c = 2$
(B) $c = \pm 2$
(C) $c = -2$
(D) $c = 0$
(E) No such value of c exists

$\boxed{f(4) = (4)^3 + 1 \quad = 64 + 1 \quad f(4) = 65}$ $\boxed{f(-2) = (-2)^3 + 1 \quad = -8 + 1 \quad f(-2) = -7}$

$ARC = \dfrac{f(4) - f(-2)}{(4) - (-2)}$
$= \dfrac{(65) - (-7)}{6}$
$= \dfrac{72}{6}$
$ARC = 12$

$f'(x) = 3x^2$

MVT
$ARC = f'(c)$
$12 = 3c^2$
$4 = c^2$
$\pm 2 = c$

The mean value theorem says a c exists between -2 and 4.
So, $-2 \notin (-2, 4)$