Find the derivative of the following.

1. $f(x) = 2\sqrt{3x^2 - 5x}$ $= 2(3x^2-5x)^{\frac{1}{2}}$

$f'(x) = 2\left(\frac{1}{2}\right)(3x^2-5x)^{-\frac{1}{2}}(3x^2-5x)'$

$= 1\cdot(3x^2-5x)^{-\frac{1}{2}}(6x-5)$

$f'(x) = \dfrac{6x-5}{\sqrt{3x^2-5x}}$

2. $g(x) = 2x(x^3 - 1)^2$

$g'(x) = (2x)'(x^3-1)^2 + (2x)[(x^3-1)^2]'$

$= (2)(x^3-1)^2 + 2x(2)(x^3-1)(x^3-1)'$

$= 2(x^3-1)^2 + 2x(2)(x^3-1)(3x^2)$

$= 2(x^3-1)[(x-1) + 2x(3x^2)]$

$= 2(x^3-1)[x-1+6x^2]$

$g'(x) = 2(x^3-1)(6x^2+x-1)$

3. $y = \dfrac{3}{(7x^2-1)^3}$ $= 3(7x^2-1)^{-3}$

$y' = -9(7x^2-1)^{-4}(7x^2-1)'$

$y' = -9(7x^2-1)^{-4}(14x)$

$y' = \dfrac{-126x}{(7x^2-1)^4}$

4. $h(x) = \dfrac{6x^2-5}{\sqrt{2-5x}}$

$h'(x) = \dfrac{(6x^2-5)'\cdot(2-5x)^{\frac{1}{2}} - (6x^2-5)\left[(2-5x)^{\frac{1}{2}}\right]'}{\left(\sqrt{2-5x}\right)^2}$

$= \dfrac{(12x)(2-5x)^{\frac{1}{2}} - (6x^2-5)\frac{1}{2}(2-5x)^{-\frac{1}{2}}(2-5x)'}{2-5x}$

$= \dfrac{(12x)(2-5x)^{\frac{1}{2}} - (6x^2-5)\frac{1}{2}(2-5x)^{-\frac{1}{2}}(-5)}{2-5x}$

$= \dfrac{(2-5x)^{-\frac{1}{2}}\left[12x(2-5x) - (6x^2-5)\frac{1}{2}(-5)\right]}{2-5x}$

$= \dfrac{24x - 60x^2 + 15x^2 - \frac{25}{2}}{(2-5x)^{\frac{1}{2}}(2-5x)}$

$h'(x) = \dfrac{-45x^2+24x-\frac{25}{2}}{\sqrt{(2-5x)^3}}$

5. $f(t) = \left(\dfrac{t^2+1}{3t}\right)^2$ $f'(t) = 2\left(\dfrac{t^2+1}{3t}\right)\cdot\left(\dfrac{t^2+1}{3t}\right)'$

$= \dfrac{2(t^2+1)}{3t}\cdot\dfrac{(t^2+1)'(3t)-(t^2+1)(3t)'}{(3t)^2}$

$= \dfrac{2(t^2+1)}{3t}\cdot\dfrac{(2t)(3t)-(t^2+1)(3)}{(3t)^2}$

$= \dfrac{2(t^2+1)}{3t}\cdot\dfrac{6t^2-3t^2-3}{(3t)^2}$

$f'(x) = \dfrac{2(t^2+1)(3t^2-3)}{(3t)^3}$

6. $f(r) = \sqrt[3]{5r^2 - 2r + 1}$ $= (5r^2-2r+1)^{\frac{1}{3}}$

$f'(r) = \frac{1}{3}(5r^2-2r+1)^{-2/3}(5r^2-2r+1)'$

$= \frac{1}{3}(5r^2-2r+1)^{-2/3}(10r-2)$

$f'(r) = \dfrac{10r-2}{3\sqrt[3]{(5r^2-2r+1)^2}}$

Find the derivatives of the following.

7. $y = x\sqrt{x-1}$ $= x\cdot(x-1)^{\frac{1}{2}}$

$\dfrac{dy}{dx} = x'(x-1)^{\frac{1}{2}} + x\left[(x-1)^{\frac{1}{2}}\right]' = 1\cdot(x-1)^{\frac{1}{2}} + x\,\frac{1}{2}(x-1)^{-\frac{1}{2}}(x-1)'$

$= (x-1)^{\frac{1}{2}} + \frac{1}{2}x(x-1)^{-\frac{1}{2}}(1)$

$= (x-1)^{-\frac{1}{2}}\left[(x-1)+\frac{1}{2}x\right]$

$\dfrac{dy}{dx} = \dfrac{\frac{3}{2}x-1}{\sqrt{x-1}}$

$\dfrac{d^2y}{dx^2} = \dfrac{(\frac{3}{2}x-1)'(x-1)^{\frac{1}{2}} - (\frac{3}{2}x-1)\left[(x-1)^{\frac{1}{2}}\right]'}{(\sqrt{x-1})^2}$

$= \dfrac{(\frac{3}{2})(x-1)^{\frac{1}{2}} - (\frac{3}{2}x-1)\frac{1}{2}(x-1)^{-\frac{1}{2}}(x-1)'}{x-1}$

$= \dfrac{\frac{3}{2}(x-1)^{\frac{1}{2}} - (\frac{3}{2}x-1)\frac{1}{2}(x-1)^{-\frac{1}{2}}(1)}{x-1}$

$= \dfrac{\frac{1}{2}(x-1)^{-\frac{1}{2}}\left[3(x-1)-(\frac{3}{2}x-1)\right]}{x-1}$

$= \dfrac{3x-3-\frac{3}{2}x+1}{2(x-1)^{\frac{1}{2}}(x-1)}$

$\dfrac{d^2y}{dx^2} = \dfrac{\frac{3}{2}x-2}{2(x-1)^{3/2}}$

8. $y = (ex^3 + 5)^{-2}$

$y' = -2(ex^3+5)^{-3}(ex^3+5)'$

$= -2(ex^3+5)^{-3}(3ex^2)$

$= -6ex^2(ex^3+5)^{-3}$

$y'' = (-6ex^2)'(ex^3+5)^{-3} + (-6ex^2)\left[(ex^3+5)^{-3}\right]'$

$= -12ex(ex^3+5)^{-3} + (-6ex^2)(-3)(ex^3+5)^{-4}(ex^3+5)'$

$= -12ex(ex^3+5)^{-3} + (-6ex^2)(-3)(ex^3+5)^{-4}(3ex^2)$

$= -6ex(ex^3+5)^{-4}\left[2(ex^3+5) + x(-3)(1)(3ex^2)\right]$

$= -6ex(ex^3+5)^{-4}\left[2ex^3+10 - 9ex^3\right]$

$y'' = \dfrac{-6ex(-7ex^3+10)}{(ex^3+5)^4}$

Evaluate the derivative at a point.

9. $f(x) = \sqrt{1 + (x^2 - 1)^3}$

$f'(2) =$

$f' = \frac{1}{2}\left[1+(x^2-1)^3\right]^{-1/2}\left[1+(x^2-1)^3\right]'$

$f' = \frac{1}{2}\left[(1+(x^2-1)^3\right]^{-1/2}\left[0 + 3(x^2-1)^2(x^2-1)'\right]$

$= \frac{1}{\cancel{2}\sqrt{1+(x^2-1)^3}}\left[3(x^2-1)^2(\cancel{2}x)\right]$

$f' = \frac{3x(x^2-1)^2}{\sqrt{1+(x^2-1)^3}}$

$f'(2) = \frac{3(2)[(2)^2-1]^2}{\sqrt{1+[(2)^2-1]^3}}$

$= \frac{6[4-1]^2}{\sqrt{1+[4-1]^3}}$

$= \frac{6(3)^2}{\sqrt{1+[3]^3}}$

$= \frac{6(9)}{\sqrt{1+27}}$

$f'(2) = \frac{54}{\sqrt{28}}$

10. $y = \frac{\pi x}{\sqrt{2x-1}}$

$\left.\frac{dy}{dx}\right|_{x=1}$

$\frac{dy}{dx} = \frac{(\pi x)'(2x-1)^{\frac{1}{2}} - (\pi x)\left[(2x-1)^{\frac{1}{2}}\right]'}{\left[\sqrt{2x-1}\right]^2}$

$= \frac{\pi(2x-1)^{\frac{1}{2}} - \pi x\left(\frac{1}{2}\right)(2x-1)^{-\frac{1}{2}}(2x-1)'}{2x-1}$

$= \frac{\pi(2x-1)^{\frac{1}{2}} - \pi x\left(\frac{1}{\cancel{2}}\right)(2x-1)^{-\frac{1}{2}}(\cancel{2})}{2x-1}$

$= \frac{\pi(2x-1)^{-\frac{1}{2}}\left[(2x-1) - x\right]}{2x-1}$

$\frac{dy}{dx} = \frac{\pi(x-1)}{(2x-1)^{3/2}}$

$\left.\frac{dy}{dx}\right|_{x=1} = \frac{\pi[(1)-1]}{[2(1)-1]^{3/2}}$

$= \frac{\pi[0]}{[2-1]^{3/2}}$

$= \frac{0}{[1]^{3/2}}$

$\left.\frac{dy}{dx}\right|_{x=1} = 0$

Write the equation of the tangent line and the normal line at the point given.

11. $f(x) = \sqrt{\frac{2x}{x+1}}$ at $x = -2$

Point $(-2, 2)$

$f(-2) = \sqrt{\frac{2(-2)}{(-2)+1}}$

$= \sqrt{\frac{-4}{-1}}$

$= \sqrt{4}$

$f(-2) = 2$

Tangent slope $m = \frac{1}{2}$

$f'(x) = \frac{1}{2}\left(\frac{2x}{x+1}\right)^{-\frac{1}{2}}\left(\frac{2x}{x+1}\right)'$

$= \frac{1}{2}\left(\frac{2x}{x+1}\right)^{-\frac{1}{2}}\frac{(2x)'(x+1) - (2x)(x+1)'}{(x+1)^2}$

$= \frac{1}{2}\left(\frac{2x}{x+1}\right)^{-\frac{1}{2}}\frac{2(x+1) - 2x(1)}{(x+1)^2}$

$= \frac{1}{2}\sqrt{\frac{x+1}{2x}} \cdot \frac{\cancel{2x}+2-\cancel{2x}}{(x+1)^2}$

$= \frac{1}{2}\sqrt{\frac{x+1}{2x}} \cdot \frac{2}{(x+1)^2}$

$f'(x) = \sqrt{\frac{x+1}{2x}} \cdot \frac{1}{(x+1)^2}$

$f'(-2) = \sqrt{\frac{(-2)+1}{2(-2)}} \cdot \frac{1}{[(-2)+1]^2}$

$= \sqrt{\frac{-1}{-4}} \cdot \frac{1}{[-1]^2}$

$= \sqrt{\frac{1}{4}} \cdot 1$

$f'(-2) = \frac{1}{2}$

Normal Slope

$\perp m = -2$

Tangent Line

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{2}(x - (-2))$

$y - 2 = \frac{1}{2}(x + 2)$

Normal line

$y - y_1 = m(x - x_1)$

$y - (2) = -2(x - (-2))$

$y - 2 = -2(x + 2)$

Particle Motion

12. The position of a particle moving along a coordinate line is $s = \sqrt{1 + 4t}$, with s in meters and t in seconds. Find the particle's velocity and acceleration at $t = 6$.

$s = (1+4t)^{\frac{1}{2}}$

1. $v = \frac{1}{2}(1+4t)^{-\frac{1}{2}} \cdot (1+4t)'$

$v = \frac{1}{2}(1+4t)^{-\frac{1}{2}}(4)$

$v = \frac{2}{\sqrt{1+4t}}$

2. $v(6) = \frac{2}{\sqrt{1+4(6)}}$

$= \frac{2}{\sqrt{1+24}}$

$= \frac{2}{\sqrt{25}}$

$v(2) = \frac{2 \text{ meters}}{5 \text{ seconds}}$

3. $v = 2(1+4t)^{-\frac{1}{2}}$

$a = -(1+4t)^{-3/2}(1+4t)'$

$a = -(1+4t)^{-3/2}(4)$

$a = \frac{-4}{\sqrt{(1+4t)^3}}$

4. $a(6) = \frac{-4}{\sqrt{(1+4(6))^3}}$

$= \frac{-4}{\sqrt{(1+24)^3}}$

$= \frac{-4}{\sqrt{(25)^3}}$

$a(6) = \frac{-4}{(5)^3}$

$a(6) = -\frac{4}{125}\ m/s^2$

Find $f'(5)$ given the following.

$g(5) = 9$ and $g'(5) = 6$

$h(5) = 5$ and $h'(5) = -4$

13. $f(x) = g(h(x))$

$f'(x) = g'(h(x)) \cdot h'(x)$

$f'(5) = g'(h(5)) \cdot h'(5)$

$= g'(5) \cdot (-4)$

$= 6(-4)$

$f'(5) = -24$

14. $f(x) = (h(x))^2$

$f'(x) = 2(h(x)) \cdot h'(x)$

$f'(5) = 2(h(5)) \cdot h'(5)$

$= 2(5) \cdot (-4)$

$f'(5) = -40$

19. $f(x) = \sqrt{g(x)}$

$f'(x) = \frac{1}{2}(g(x))^{-\frac{1}{2}} \cdot g'(x)$

$f'(5) = \frac{1}{2\sqrt{g(5)}} \cdot g'(5)$

$= \frac{1}{2\sqrt{9}} \cdot (6)$

$= \frac{3}{3}$

$f'(5) = 1$

20. $f(x) = 2g(x)h(x)$

$f'(x) = 2g'(x) \cdot h(x) + 2g(x) \cdot h'(x)$

$f'(5) = 2g'(5) \cdot h(5) + 2g(5) \cdot h'(5)$

$= 2 \cdot (6)(5) + 2(9) \cdot (-4)$

$= 60 - 72$

$f'(5) = -12$

MULTIPLE CHOICE

1. Let $f(x) = x \cdot g(h(x))$ where $g(4) = 2, g'(4) = 3, h(3) = 4,$ and $h'(3) = -2$. $\boxed{\text{Find } f'(3).}$

   (A) -18
   (B) -16
   (C) -7
   (D) 7
   (E) 11

$f'(x) = x' \cdot g(h(x)) + x \cdot [g(h(x))]'$

$= 1 \cdot g(h(x)) + x \cdot g'(h(x)) \cdot h'(x)$

$f'(3) = g(h(3)) + (3) \cdot g'(h(3)) \cdot h'(3)$

$= g(4) + 3g'(4) \cdot (-2)$

$= 2 + 3 \cdot (3)(-2)$

$= 2 - 18$

$f'(3) = -16$

2. Let m and b be real numbers and let the function f be defined by

$$f(x) = \begin{cases} 1 + 3bx + 2x^2 & \text{for } x \leq 1 \\ mx + b & \text{for } x \leq 1 \end{cases}$$

   If f is both continuous and differentiable at $x = 1$, then

   (A) $m = 1, b = 1$
   (B) $m = 1, b = -1$
   (C) $m = -1, b = 1$
   (D) $m = -1, b = -1$
   (E) none of the above

$1 + 3bx + 2x^2 = mx + b$

@ $x = 1$: $1 + 3b(1) + 2(1)^2 = m(1) + b$

$1 + 3b + 2 = m + b$

$3b + 3 = m + b$

$2b + 3 = m$

3. A particle moves on the x-axis with position defined by: $x(t) = t^3 - 6t^2 + 2t + 1$ where $t \geq 0$. What is the velocity of the particle when its acceleration is zero?

   (A) -11
   (B) -10
   (C) -1
   (D) 2
   (E) 50

① $v = 3t^2 - 12t + 2$

② $a = 6t - 12$

when is acceleration zero?

③ $0 = 6t - 12$

$12 = 6t$

$2 = t$

④ $v(2) = 3(2)^2 - 12(2) + 2$

$= 3(4) - 24 + 2$

$= 12 - 22$

$v(2) = -10$ units/time

4. If $f(x) = \sqrt{1 + \sqrt{x}}$, find $f'(x)$.

   (A) $\frac{-1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$
   (B) $\frac{1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$
   (C) $\frac{1}{4\sqrt{1+\sqrt{x}}}$
   (D) $\frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}$
   (E) $\frac{-1}{2\sqrt{x}\sqrt{1+\sqrt{x}}}$

$f'(x) = \frac{1}{2}(1 + \sqrt{x})^{-\frac{1}{2}}(1 + x^{\frac{1}{2}})'$

$= \frac{1}{2}(1 + \sqrt{x})^{-\frac{1}{2}}(0 + \frac{1}{2}x^{-\frac{1}{2}})$

$= \frac{1}{2}(1 + \sqrt{x})^{-\frac{1}{2}}(\frac{1}{2}x^{-\frac{1}{2}})$

$= \frac{1}{4\sqrt{1+\sqrt{x}}\,\sqrt{x}}$

You are allowed to use a graphing calculator for #5

5. If $(x) = \left(1+\frac{x}{20}\right)^5$, find $f''(40)$.

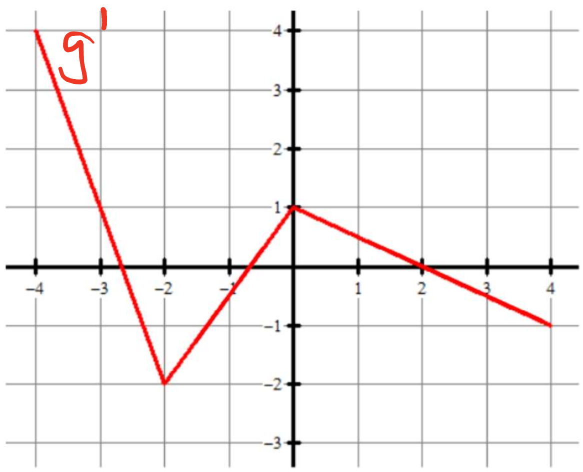
(A) 0.068
(B) 1.350
(C) 5.400
(D) 6.750
(E) 540.000

$Y_1 = \left(1+\frac{x}{20}\right)^5$
$Y_2 = \text{nderiv}(Y_1, x, x)$
$Y_2 = \text{nderiv}(Y_2, x, 40)$

FREE RESPONSE

Your score: ____ out of 4

1. The graph of the function *f*, shown below, consists of three line segments. Suppose $g(x)$ is a function whose derivative is *f*.



Graph of *f*

(a) Suppose $y = x + 7$ is the equation for the line tangent to the graph of $g(x)$ at $x = -3$. Let *h* be the function defined by $h(x) = (g(x))^2$. Find $h'(-3)$.

$g'(x) = x+7$ @ $x = -3$

$g'(-3) = -3+7$
$g'(-3) = 4$

$h'(x) = 2(g(x))^1 \cdot g'(x)$
$h'(-3) = 2 \cdot g(-3) \cdot g'(-3)$
$= 2 \cdot (4) \cdot (1)$
$h'(3) = 8$

(b) Describe the shape of the graph of $g(x)$ near $x = 2$.

As x approaches 2 from the left, the derivative is positive meaning the function is increasing.
As x approaches 2 from the right, the derivative is negative meaning the function is decreasing.
At $x = 2$, the derivative is zero which means the slope of the tangent line is zero causing a maximum or minimum point. Since the function is increasing and then decreasing it must be a maximum point.

(c) Give a piecewise defined equation for $g''(x)$.

$$f(x) = \begin{cases} -3 & -4 < x < -2 \\ \frac{3}{2} & -2 < x < 0 \\ -\frac{1}{2} & 0 < x < 4 \end{cases}$$