3.4 Chain Rule

NOTES

CALCULUS

Write your questions here!

Find the derivative.

$$f(x) = (3x^4 - 2x + 5)^3$$

$$f'(x) = 3(3x^4-2x+5)^2(3x^4-2x+5)'$$
$$= 3(3x^4-2x+5)^2(12x^3-2)$$

CHAIN RULE

$$\frac{d}{dx}f(g(x)) = f'(g)\cdot g'$$

Find the derivative of the following.

$$f(x) = \sqrt{4x-3}$$

$$f(x) = (4x-3)^{\frac{1}{2}}$$
$$f'(x) = \frac{1}{2}(4x-3)^{-\frac{1}{2}}(4x-3)'$$
$$= \frac{1}{2}(4x-3)^{-\frac{1}{2}}(4)$$
$$f'(x) = \frac{2}{\sqrt{4x-3}}$$

$$y = 2x\sqrt[3]{2x^4-9}$$

$$y = 2x^3(2x^4-9)^{\frac{1}{3}}$$

Product Rule!

$$y' = (2x^3)'(2x^4-9)^{\frac{1}{3}} + (2x^3)\left[(2x^4-9)^{\frac{1}{3}}\right]'$$
$$y' = 6x^2(2x^4-9)^{\frac{1}{3}} + 2x^3\left(\frac{1}{3}\right)(2x^4-9)^{-2/3}(2x^4-9)'$$
$$y' = 6x^2(2x^4-9)^{\frac{1}{3}} + 2x^3\left(\frac{1}{3}\right)(2x^4-9)^{-2/3}(8x^3)$$

FACTOR GCF

$$y' = 2x^2(2x^4-9)^{-2/3}\left[3(2x^4-9) + x\left(\frac{1}{3}\right)\right]$$
$$y' = 2x^2(2x^4-9)^{-2/3}\left[6x^4 - 27 + \frac{1}{3}x\right]$$
$$y' = \frac{2x^2(6x^4 - \frac{1}{3}x - 27)}{(\sqrt[3]{2x^4-9})^2}$$

Evaluate

$$f(x) = \frac{1}{\sqrt{3x-5}} = (3x-5)^{-\frac{1}{2}}$$

Find $f'(7)$

$$f'(x) = -\frac{1}{2}(3x-5)^{-3/2}(3x-5)'$$
$$= -\frac{1}{2}(3x-5)^{-3/2}(3)$$
$$f'(x) = \frac{-3}{2(\sqrt{3x-5})^3}$$

$$f'(7) = \frac{-3}{2(\sqrt{3(7)-5})^3}$$
$$= \frac{-3}{2(\sqrt{21-5})^3}$$
$$= \frac{-3}{2(\sqrt{16})^3}$$
$$= \frac{-3}{2(4)^3}$$
$$= \frac{-3}{2\cdot 64}$$
$$f'(7) = \frac{-3}{128}$$

Find the derivative.

$$f(x) = \frac{\sqrt{3x+1}}{2x+1} = \frac{(3x+1)^{\frac{1}{2}}}{2x+1}$$

$$f'(x) = \frac{\left[(3x+1)^{\frac{1}{2}}\right]'(2x+1) - (3x+1)^{\frac{1}{2}}(2x+1)'}{(2x+1)^2}$$

$$= \frac{\frac{1}{2}(3x+1)^{-\frac{1}{2}}(3x+1)'(2x+1) - (3x+1)^{\frac{1}{2}}(2)}{(2x+1)^2}$$

$$f'(x) = \frac{\frac{1}{2}(3x+1)^{-\frac{1}{2}}(3)(2x+1) - (3x+1)^{\frac{1}{2}}(2)}{(2x+1)^2}$$

$$f'(x) = \frac{(3x+1)^{-\frac{1}{2}}\left[\frac{1}{2}(3)(2x+1) - (3x+1)(2)\right]}{(2x+1)^2}$$

$$= \frac{(3x+1)^{-\frac{1}{2}}\left[3x + \frac{3}{2} - 6x + \frac{4}{2}\right]}{(2x+1)^2}$$

$$f'(x) = \frac{-3x + \frac{7}{2}}{\sqrt{3x+1}\,(2x+1)^2}$$

Find the derivative.

$$f(x) = \left(\frac{t^2+1}{(3t-1)^2}\right)^3$$

$$f'(t) = 3\left(\frac{t^2+1}{(3t-1)^2}\right)^2 \cdot \left(\frac{t^2+1}{(3t-1)^2}\right)'$$

$$= 3\,\frac{(t^2+1)^2}{(3t-1)^4} \cdot \frac{(t^2+1)'(3t-1)^2 - (t^2+1)\left[(3t-1)^2\right]'}{\left[(3t-1)^2\right]^2}$$

$$= 3\,\frac{(t^2+1)^2}{(3t-1)^4} \cdot \frac{2t\cdot(3t-1)^2 - (t^2+1)\,2(3t-1)(3t-1)'}{(3t-1)^4}$$

$$= 3\,\frac{(t^2+1)^2}{(3t-1)^4} \cdot \frac{2t(3t-1)^2 - (t^2+1)2(3t-1)(3)}{(3t-1)^4}$$

$$= 3\,\frac{(t^2+1)^2}{(3t-1)^4} \cdot \frac{2(3t-1)\left[t(3t-1) - (t^2+1)(3)\right]}{(3t-1)^4}$$

$$= 3\,\frac{(t^2+1)^2}{(3t-1)^4} \cdot \frac{2(3t-1)\left[3t^2 - t - 3t^2 - 3\right]}{(3t-1)^4}$$

$$= 3\,\frac{(t^2+1)^2}{(3t-1)^4} \cdot \frac{2(3t-1)(-t-3)}{(3t-1)^4}$$

$$= \frac{-6(t^2+1)^2(3t-1)(t+3)}{(3t-1)^8}$$

$$f'(t) = \frac{-6(t^2+1)^2(t+3)}{(3t-1)^7}$$

Find $f'(4)$ given the following:

$$g(4) = 3 \text{ and } g'(4) = -2$$

$$h(4) = 9 \text{ and } h'(4) = 5$$

$f(x) = (g(x))^2$

$$f'(x) = 2(g(x)) \cdot g'(x)$$

$$f'(4) = 2(g(4)) \cdot g'(4)$$

$$= 2(3)\cdot(-2)$$

$$f'(4) = -12$$

$f(x) = \sqrt{h(x)}$

$$f(x) = [h(x)]^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}[h(x)]^{-\frac{1}{2}}\,h'(x)$$

$$f'(x) = \frac{h'(x)}{2\sqrt{h(x)}}$$

$$f'(4) = \frac{h'(4)}{2\sqrt{h(4)}}$$

$$= \frac{5}{2\sqrt{9}}$$

$$= \frac{5}{2\cdot 3}$$

$$f'(4) = \frac{5}{6}$$

$f(x) = h(g(x))$

$$f'(x) = h'(g(x)) \cdot g'(x)$$

$$f'(4) = h'(g(4)) \cdot g'(4)$$

$$= h'(3) \cdot (-2)$$

$$f'(4) = -2h'(3)$$

SUMMARY:

Now, summarize your notes here!