# PRACTICE

**Warm Up! Find the derivative of the following.**

1. $y = \cos(2x)$

$\frac{dy}{dx} = -\sin(2x)\cdot(2x)'$

$\frac{dy}{dx} = -2\sin 2x$

2. $f(x) = 2\sin x$

$f'(x) = 2\cos x$

3. $y = \cos^2 x = [\cos(x)]^2$

$y' = 2\cos(x)\cdot\cos'(x)$

$y = -2\cos(x)\sin(x)$

4. $f(x) = \csc(\pi x)$

$\frac{df}{dx} = -\csc(\pi x)\cot(\pi x)\cdot(\pi x)'$

$\frac{df}{dx} = -\pi\cdot\csc(\pi x)\cot(\pi x)$

5. $y = -3\tan(5x^3)$

$\dot{y} = -3\sec^2(5x^3)\cdot(\dot{5x^3})$

$= -3\sec^2(5x^3)\cdot 15x^2$

$\dot{y} = -45x^2\sec^2(5x^3)$

6. $f(\theta) = 5\sec(4\theta)$

$\dot{f}(\theta) = 5\sec(4\theta)\tan(4\theta)\cdot(\dot{4\theta})$

$\dot{f}(\theta) = 20\sec(4\theta)\tan(4\theta)$

**Warm Up! Evaluate the derivative at a point.**

7. $f(x) = 3\sin(2x)$

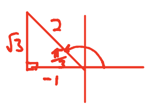
$f'(x) = 3\cos(2x)\cdot(2x)'$

$f'(x) = 6\cos(2x)$

$f'(\frac{\pi}{3}) = 6\cos[2(\frac{\pi}{3})]$

$= 6\cos\frac{2\pi}{3}$

$= 6(-\frac{1}{2})$




$f'\left(\frac{\pi}{3}\right) = -3$

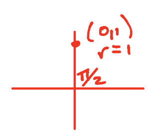
8. $f(\theta) = -2\csc\theta + 4$

$f'(\theta) = +2\csc\theta\cot\theta$

$f'(\frac{\pi}{2}) = 2\csc(\frac{\pi}{2})\cot(\frac{\pi}{2})$

$= 2\cdot 1\cdot 0$

$f'(\frac{\pi}{2}) = 0$




$f'\left(\frac{\pi}{2}\right) = 0$

9. $y = \cos^2 x = [\cos(x)]^2$

$y' = 2\cos(x)\cdot[\cos(x)]'$

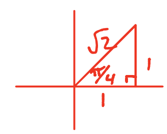
$y' = 2\cos(x)(-\sin(x))$

$y' = -2\cos(x)\sin(x)$

$\frac{dy}{dx}\Big|_{x=\frac{\pi}{4}} = -2\cos(\frac{\pi}{4})\sin(\frac{\pi}{4})$

$= -2(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2})$

$= \frac{-2\cdot 2}{2\cdot 2}$




$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = -1$

**Find the derivative of the following.**

10. $f(x) = 2\sin x + \cos x$

$f'(x) = 2\cos x - \sin x$

11. $g(x) = 2x(\cos 4x)^2$

u v

Chain Rule

chain Rule

$g'(x) = (2x)'(\cos(4x))^2 + (2x)[(\cos(4x))^2]'$

$= 2\cos^2(4x) + 2x(2)\cos(4x)\cdot[\cos(4x)]'$

$= 2\cos^2(4x) + 4x\cos(4x)\cdot(-\sin(4x))\cdot(4x)'$

$= 2\cos^2(4x) - 4x\cos(4x)\sin(4x)\cdot 4$

$= 2\cos^2(4x) - 16x\cos(4x)\sin(4x)$

12. $y = 5 - \csc\left(\frac{x^2}{2}\right)$

Chain

$\frac{dy}{dx} = 0 - \left[-\csc(\frac{x^2}{2})\cot(\frac{x^2}{2})\right]\cdot(\frac{1}{2}x^2)'$

$= \csc(\frac{x^2}{2})\cot(\frac{x^2}{2})\cdot x$

$\frac{dy}{dx} = x\csc(\frac{x^2}{2})\cot(\frac{x^2}{2})$

13. $h(x) = 2x\tan(5x)$

u v

Chain Rule

$h'(x) = (2x)'\tan(5x) + 2x[\tan(5x)]'$

$= 2\tan(5x) + 2x\sec^2(5x)\cdot(5x)'$

$= 2\tan(5x) + 2x\sec^2(5x)\cdot(5)$

$= 2\tan(5x) + 10x\sec^2(5x)$

14. $f(x) = \frac{1}{2\sin x} = \frac{1}{2}[\sin(x)]^{-1}$

$f'(x) = -\frac{1}{2}[\sin(x)]^{-2}[\sin(x)]'$

$= -\frac{1}{2}\sin^{-2}(x)\cos(x)$

$f'(x) = \frac{-\cos(x)}{2\sin^2(x)}$

15. $y = \sec(\pi x + 1)$

$y' = \sec(\pi x+1)\tan(\pi x+1)\cdot(\pi x+1)'$

$y' = \pi\sec(\pi x+1)\tan(\pi x+1)$

16. $r = \sqrt{\theta \sin\theta} = [\theta\sin\theta]^{\frac{1}{2}}$

$r' = \frac{1}{2}[\theta\sin\theta]^{-\frac{1}{2}}[\theta\cdot\sin\theta]'$

$r' = \frac{1}{2\sqrt{\theta\sin\theta}}\cdot(\theta'\cdot\sin\theta + \theta\cdot\sin'\theta)$

$r' = \frac{1}{2\sqrt{\theta\sin\theta}}\cdot(1\cdot\sin\theta + \theta\cos\theta)$

$r' = \frac{\sin\theta+\theta\cos\theta}{2\sqrt{\theta\sin\theta}}$

17. $s = \overset{u}{t}\,\overset{v}{\cos(\pi - 4t)}$

Chain

$s' = t'\cos(\pi-4t) + t\cdot\cos'(\pi-4t)$

$s' = 1\cdot\cos(\pi-4t) + t(-\sin(\pi-4t))\cdot(\pi-4t)'$

$s' = \cos(\pi-4t) + 4t\sin(\pi-4t)$

## Evaluate the derivative at a point.

18. $f(x) = \cos(\tan x)$

chain

$f'(x) = -\sin(\tan x)\cdot\tan' x$

$f'(x) = -\sin(\tan x)\cdot\sec^2 x$

$f'(\pi) = -\sin(\tan\pi)\cdot\sec^2\pi$

$= -\sin(0)\cdot(-1)^2$

$= 0\,(1)$

$f'(\pi) = 0$

$f'(\pi) =$ 0

19. $y = \frac{\sin x}{1-\cos 2x}$

$y' = \frac{\sin' x\cdot(1-\cos(2x)) - \sin x(1-\cos(2x))'}{[1-\cos 2x]^2}$

$y' = \frac{\cos x(1-\cos 2x) - \sin x(0+\sin(2x)\cdot(2x)')}{[1-\cos 2x]^2}$

$y' = \frac{\cos x(1-\cos 2x) - 2\sin x\sin 2x}{(1-\cos 2x)^2}$

$y'(\frac{\pi}{4}) = \frac{\cos(\frac{\pi}{4})(1-\cos 2(\frac{\pi}{4})) - 2\sin(\frac{\pi}{4})\sin 2(\frac{\pi}{4})}{(1-\cos 2(\frac{\pi}{4}))^2}$

$= \frac{\frac{\sqrt{2}}{2}(1-0) - 2\frac{\sqrt{2}}{2}\cdot 1}{(1-0)^2}$

$= \frac{\frac{\sqrt{2}}{2}-\sqrt{2}}{1^2}$

$y'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}-\sqrt{2}$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \frac{\sqrt{2}}{2}-\sqrt{2}$

## Write the equation of the tangent line and the normal line at the point given.

20. $f(x) = \tan^2 x$ at $x = \frac{\pi}{4}$

Point $(\frac{\pi}{4}, 1)$

$f(\frac{\pi}{4}) = \tan^2(\frac{\pi}{4})$

$= (1)^2$

$f(\frac{\pi}{4}) = 1$

Slope tangent $m=4$

$f'(x) = 2\tan(x)\cdot\tan'(x)$

$f'(x) = 2\tan(x)\cdot\sec^2(x)$

$f'(\frac{\pi}{4}) = 2\tan(\frac{\pi}{4})\sec^2(\frac{\pi}{4})$

$= 2(1)\cdot(\sqrt{2})^2$

$= 2\cdot 2$

$f'(\frac{\pi}{4}) = 4$

Normal slope

$\perp m = -\frac{1}{4}$

tangent line

$y-y_1 = m(x-x_1)$

$y-1 = 4(x-\frac{\pi}{4})$

Normal line

$y-y_1 = m(x-x_1)$

$y-1 = -\frac{1}{4}(x-\frac{\pi}{4})$

## Particle Motion

21. The position of a particle moving along a coordinate line is $s(t) = 2\sin\pi t + 5\cos\pi t$, with $s$ in meters and $t$ in seconds. Find the particle's velocity and acceleration at $t = 1$.

$v = 2\cos(\pi t)\cdot(\pi t)' - 5\sin(\pi t)\cdot(\pi t)'$

$v = 2\pi\cos(\pi t) - 5\pi\sin(\pi t)$

$v(1) = 2\pi\cos(\pi\cdot 1) - 5\pi\sin(\pi\cdot 1)$

$= 2\pi(-1) - 5\pi(0)$

$v(1) = -2\pi$ m/s

$2\pi$ m/s left

$v = 2\pi\cos(\pi t) - 5\pi\sin(\pi t)$

$a = -2\pi\sin(\pi t)(\pi t)' - 5\pi\cos(\pi t)\cdot(\pi t)'$

$a = -2\pi^2\sin(\pi t) - 5\pi^2\cos(\pi t)$

$a(1) = -2\pi\sin(\pi\cdot 1) - 5\pi^2\cos(\pi\cdot 1)$

$= -2\pi(0) - 5\pi^2(-1)$

$a(1) = 5\pi^2$ m/s²

## MULTIPLE CHOICE

1. If $f(x) = \dfrac{\sin\sqrt{x}}{\sqrt{x}}$, then $f'(x)$ is

   (A) $\dfrac{\cos\sqrt{x}}{2x} - \dfrac{\sin\sqrt{x}}{2\sqrt{x^3}}$

   (B) $\dfrac{\cos\sqrt{x} - \sin\sqrt{x}}{2x}$

   (C) $\dfrac{\sqrt{x}\cos\sqrt{x} - \frac{\sin\sqrt{x}}{2\sqrt{x}}}{x}$

   (D) $\cos\sqrt{x}$

   (E) $\dfrac{\frac{\cos\sqrt{x}}{2} + \frac{\sin\sqrt{x}}{2\sqrt{x}}}{x}$

$f'(x) = \dfrac{(\sin\sqrt{x})'\cdot\sqrt{x} - (\sin\sqrt{x})(\sqrt{x})'}{(\sqrt{x})^2}$

$= \dfrac{\cos(\sqrt{x})\cdot(\sqrt{x})'\cdot\sqrt{x} - \sin\sqrt{x}\cdot\frac{1}{2}x^{-\frac{1}{2}}}{x}$

$= \dfrac{\cos\sqrt{x}\cdot(\frac{1}{2})x^{-\frac{1}{2}}\sqrt{x} - \frac{\sin\sqrt{x}}{2\sqrt{x}}}{x}$

$= \dfrac{\left(\frac{\cos\sqrt{x}}{2} - \frac{\sin\sqrt{x}}{2\sqrt{x}}\right)\frac{1}{x}}{x\,\frac{1}{x}}$

$= \dfrac{\cos\sqrt{x}}{2x} - \dfrac{\sin\sqrt{x}}{2\sqrt{x^3}}$

2. What is $\lim\limits_{h\to 0}\dfrac{\cos\left(\frac{\pi}{2}+h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$ ?

   Def'n of derivative

   (A) $-1$

   (B) $-\dfrac{\sqrt{2}}{2}$

   (C) 0

   (D) 1

   (E) The limit does not exist.

$\dfrac{d}{dx}[\cos(x)]\Big|_{\pi/2} = -\sin(x)\Big|_{\pi/2} = -\sin\left(\frac{\pi}{2}\right) = -1$

**You are allowed to use a graphing calculator for 3-6**

3. Let $f(x) = \sqrt{2x}$. If the rate of change of $f$ at $x = c$ is four times its rate of change at $x = 1$, then $c =$

   (A) $\dfrac{1}{16}$

   (B) $\dfrac{1}{2\sqrt{2}}$

   (C) $\dfrac{1}{\sqrt{2}}$

   (D) 1

   (E) 32

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(1) = \frac{1}{2\sqrt{1}}$

$f'(1) = \frac{1}{2}$

c is 4 times

$4\cdot\frac{1}{2} = 2$ ...

$f'(x) = \frac{1}{2\sqrt{x}}$

$2 = \frac{1}{2\sqrt{c}}$

$4 = \frac{1}{\sqrt{c}}$

$4\sqrt{c} = 1$

$\sqrt{c} = \frac{1}{4}$

$c = \frac{1}{16}$

4. Which of the following is an equation of the line tangent to the graph of $f(x) = x^6 + x^4$ at the point where $f'(x) = -1$?

   (A) $y = -x - 1.031$

   (B) $y = -x - 0.836$

   (C) $y = -x + 0.836$

   (D) $y = -x + 0.934$

   (E) $y = -x + 1.031$

Slope

$f' = 6x^5 + 4x^3$

$-1 = 6x^5 + 4x^3$

use calc.

$(-0.555, -1)$

x-value   slope

Point

$f(-.555) = .124$

Point-Slope

$y - y_1 = m(x - x_1)$

$y - (.124) = -1(x - (-.555))$

$y - .124 = -x - .555$

$y = -x - .431$

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5. If $f(x) = -\frac{1}{|x|}$, then $f'(2) =$

(A) 0.050
(B) $-0.250$
(C) 0.250
(D) $-0.050$
(E) $-0.500$

$Y_1 = -1/\text{abs}(x)$

2nd calc dy/dx at x=2

6. At time $t \geq 0$, the position of a particle moving along the $x$-axis is given by $x(t) = \frac{t^3}{3} + 2t + 2$. For what value of $t$ in the interval $[0,3]$ will the instantaneous velocity of the particle equal the average velocity of the particle from time $t = 0$ to time $t = 3$

(A) 1
(B) $\sqrt{3}$
(C) $\sqrt{7}$
(D) 3
(E) 5

$x(0) = \frac{(0)^3}{3} + 2(0) + 2$
$x(0) = 2$

$x(3) = \frac{(3)^3}{3} + 2(3) + 2$
$= \frac{27}{3} + 6 + 2$
$= 9 + 8$
$x(3) = 17$

ARC

$ARC = \frac{x(3) - x(0)}{3 - 0}$
$= \frac{(17) - (2)}{3}$
$= \frac{15}{3}$
$ARC = 5$

Derivative

$x'(t) = t^2 + 2$

MVT

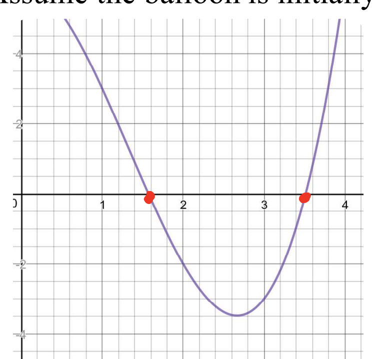
$ARC = x'(c)$
$5 = c^2 + 2$
$3 = c^2$
$\pm\sqrt{3} = c$

**You are allowed to use a graphing calculator on the Free Response**

## FREE RESPONSE

**Your score: ____ out of 5**

1. The rate of change, in kilometers per hour, of the altitude of a hot air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for time $0 \leq t \leq 4$, where $t$ is measured in hours. Assume the balloon is initially at ground level.




(a) For what values of $t$, $0 \leq t \leq 4$, is the altitude of the balloon decreasing?

$1.572 < t < 3.514$

(b) Find the value of $r'(2)$ and explain the meaning of the answer in the context of the problem. Indicate units of measure.

acceleration

$r'(t) = 3t^2 - 8t$
$r'(2) = 3(2)^2 - 8(2)$
$= 3(4) - 16$
$= 12 - 16$
$r'(2) = -4 \text{ km/hr}^2$

At 2 hours, the velocity of the balloon is decreasing by 4 km/hr each hour

(c) When does the hot air balloon have an acceleration of zero? Justify.

$0 = 3t^2 - 8t$
$0 = t(3t - 8)$
$0 = t$
$0 = 3t - 8$
$8 = 3t$
$\frac{8}{3} = t$

Since $r'(t)$ is acceleration, set $r'(t) = 0$ and solve.