UNIT 3 Basic Differentiation

NAME:____________________________

# REVIEW

DATE:_______

**Evaluate the limit.**

1. $\lim\limits_{x\to\infty} \dfrac{x^3+5x^2-x}{1-e^x} = 0$

fast

faster

2. $\lim\limits_{x\to 2} \dfrac{x^2+7x-18}{x^2-2x} = \lim\limits_{x\to 2} \dfrac{(x+9)(x-2)}{x(x-2)} = \lim\limits_{x\to 2} \dfrac{x+9}{2}$

$= \dfrac{(2)+9}{2} = \dfrac{11}{2}$

**Given $f(x)$ on a given interval $[a, b]$, find a value $c$ that satisfies the Mean Value Theorem.**

3. $f(x) = -x^2 + 4x - 2;\ [-1, 2]$

$f'(x) = -2x+4$

$ARC = \dfrac{f(b)-f(a)}{b-a} = \dfrac{f(2)-f(-1)}{(2)-(-1)} = \dfrac{2-(-7)}{3} = \dfrac{9}{3}$

$ARC = 3$

$\boxed{\begin{aligned} f(2) &= -(2)^2+4(2)-2 \\ &= -(4)+8-2 \\ f(2) &= 2 \end{aligned}}$

$\boxed{\begin{aligned} f(-1) &= -(-1)^2+4(-1)-2 \\ &= -(1)-4-2 \\ f(-1) &= -7 \end{aligned}}$

MVT

$\boxed{\begin{aligned} ARC &= f'(c) \\ 3 &= -2(c)+4 \\ -1 &= -2c \\ \tfrac{1}{2} &= c \end{aligned}}$

**Find $b$ and $c$ so that $f(x)$ is differentiable at $x = 1$.**

4. $f(x) = \begin{cases} 3x^2 + 4x, & x \le 1 \\ 2x^3 + bx + c, & x > 1 \end{cases}$

Continuous?

$3x^2+4x = 2x^3+bx+c$

at $x=1$: $3(1)^2+4(1) = 2(1)^3+b(1)+c$

$3(1)+4 = 2(1)+b+c$

$7 = 2+b+c$

$5 = b+c$

differentiable?

$(3x^2+4x)' = (2x^3+bx+c)'$

$6x+4 = 6x^2+b$

at $x=1$: $6(1)+4 = 6(1)^2+b$

$10 = 6+b$

$4 = b$

$5 = b+c$

$5 = (4)+c$

$1 = c$

$4 = b$

$b = 4$

$c = 1$

**Find the derivative of the following.**

5. $f(x) = \dfrac{\sin x}{x^2+1}$

$f'(x) = \dfrac{\sin'(x)\cdot(x^2+1) - \sin(x)(x^2+1)'}{(x^2+1)^2}$

$f'(x) = \dfrac{\cos(x)(x^2+1) - 2x\sin(x)}{(x^2+1)^2}$

6. $g(x) = \sqrt{2x^3 - 4x}$

$\dfrac{dg}{dx} = \frac{1}{2}(2x^3-4x)^{-\frac{1}{2}}\dfrac{d}{dx}(2x^3-4x)$

$= \dfrac{1}{2\sqrt{2x^3-4x}}(6x^2-4)$

$\dfrac{dg}{dx} = \dfrac{6x^2-4}{2\sqrt{2x^3-4x}}$

7. $y = \dfrac{x^3+4x-1}{2x}$

$y = \frac{1}{2}x^2 + 2 - \frac{1}{2}x^{-1}$

$\dfrac{dy}{dx} = x + \frac{1}{2}x^{-2}$

$\dfrac{dy}{dx} = x + \dfrac{1}{2x^2}$

8. $h(x) = \cos^2(4x) = [\cos(4x)]^2$

$h'(x) = 2\cos(4x)\cdot[\cos(4x)]'$

$= 2\cos(4x)[-\sin(4x)]\cdot(4x)'$

$h'(x) = -8\cos(4x)\sin(4x)$

**Find the following**

9. $f(x) = x^2 \sin(x)$

$f\left(\frac{\pi}{2}\right) =$

$f'(x) = (x^2)' \sin(x) + x^2 [\sin(x)]'$

$f'(x) = 2x\sin(x) + x^2\cos(x)$

$f'(\frac{\pi}{2}) = 2(\frac{\pi}{2})\sin(\frac{\pi}{2}) + (\frac{\pi}{2})^2\cos(\frac{\pi}{2})$

$= \pi \cdot (1) + \frac{\pi^2}{4}(0)$

$f'(\frac{\pi}{2}) = \pi$

10. $g(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$g''(x) =$

$g'(x) = -\frac{1}{2}x^{-3/2}$

$g''(x) = \frac{3}{4}x^{-5/2}$

$g''(x) = \frac{3}{4\sqrt{x^5}}$

**Write the equation of the tangent line and the normal line at the point given.**

11. $f(x) = 3\tan x$ at $x = \pi$

Point $(\pi, 0)$

$f(\pi) = 3\tan(\pi)$

$= 3 \cdot 0$

$f(\pi) = 0$

tangent Slope $m=3$

$f'(x) = 3\sec^2(x)$

$f'(\pi) = 3\sec^2(\pi)$

$= 3(-1)^2$

$= 3(1)$

$f'(\pi) = 3$

Normal Slope

$\perp m = -\frac{1}{3}$

tangent line

$y - y_1 = m(x - x_1)$

$y - (0) = 3(x - \pi)$

$y = 3(x - \pi)$

Normal line

$y - y_1 = m(x - x_1)$

$y - (0) = -\frac{1}{3}(x - \pi)$

$y = -\frac{1}{3}(x - \pi)$

**Particle Motion**

12. The position of a particle moving along a coordinate line is $s(t) = 2t^3 - 6t$ , with $s$ in meters and $t$ in seconds. Find the particle's velocity and acceleration at $t = 6$.

S = meters

t = seconds

$S(t) = 2t^3 - 6t$

$V(t) = 6t^2 - 6$

$a(t) = 12t$

$V(6) = 6(6)^2 - 6$

$= 6(36) - 6$

$= 216 - 6$

$V(6) = 210$ m/s

$a(6) = 12(6)$

$a(6) = 72$ m/s$^2$

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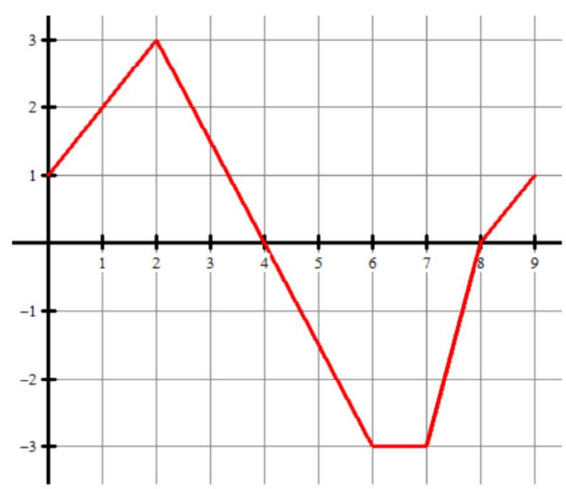
13. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.




a) When does the body reverse direction?

(when the velocity becomes negative) 4 seconds

b) When is the body moving at a constant speed?

(when velocity is horizontal) 6 to 7 seconds

c) What is the body's maximum speed?

(Absolute value of Max) 3 meters per second

d) At what time interval(s) is the body slowing down?

(when graph gets closer to x-axis) 2 to 4 seconds and 7 to 8 seconds

**Use the information to find the following.**

14. The table shows the number of stores of a popular US coffee chain from 2000 to 2006. The number of stores recorded is the number at the start of each year, on January 1st.

| ***t*** **(year)** | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
|---|---|---|---|---|---|---|---|
| ***S*** **(stores)** | 1996 | 2729 | 3501 | 4272 | 5239 | 6177 | 7353 |

Approximate the instantaneous rate of change in coffee stores per year at the beginning of 2003.

$S'(2003) \approx ARC(2002, 2004) = \frac{S(2004) - S(2002)}{(2004) - (2002)}$

$= \frac{(5239) - (3501)}{2}$

$= \frac{1738}{2}$

$S'(2003) \approx 869$ Stores/year

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## You are allowed to use a graphing calculator for #15

15. The amount $A(t)$ of pain reliever in milligrams in a patient's system after $t$ minutes is given by $A(t) = 8te^{-t/50}$.

a. Find $A(60)$. Explain what it means in a sentence.

$A(60) = 144.573$ mg

minutes

Sixty minutes after a pain reliever is given, the amount of pain reliever in a patient's system is 144.573 mg.

b. Find $A'(60)$. Explain what it means in a sentence.

$A'(60) = -.482$ mg/min

minutes

Sixty minutes after a pain reliever is given, the amount of pain reliever in a patient's system is decreasing by .482 mg/min

c. Find $A(t) = 100$. Explain what it means in a sentence.

$A(107.665) = 100$

$A(17.870) = 100$

At 17.870 minutes and 107.665 minutes after a pain reliever is given, the amount of pain reliever in a patient's system is 100 mg

d. What is the average rate of change of students from 60 minutes to 180 minutes?

$A(60) = 144.573222$ mg

$A(180) = 39.346160$ mg

$ARC = \frac{A(180) - A(60)}{(180) - (60)}$ mg/min

$= \frac{39.346160 - 144.573222}{120}$

$= \frac{-105.227062}{120}$

$ARC \approx -.877$ mg/min

e. What is the instantaneous rate of change at 180 minutes?

$A'(180) = -.568$

f. When does $A'(t) = 0$ ? What is happening at this point?

$A'(50) = 0$

when $A'(t) = 0$, this is a horizontal tangent.

At 50 minutes, the maximum amount of pain reliever is in the patient's system.

g. Find $\lim_{t \to \infty} A(t)$. Explain what it means in a sentence.

As time continues forever, the pain reliever in the patient's system approaches 0.

# TEST PREP

1. A particle is traveling along the $x$-axis. Its position is given by $x(t) = \frac{1-t^2}{t+3}$ at time $t \geq 0$. Find the instantaneous rate of change of $x$ with respect to $t$ when $t = 1$.

   (A) $-2$
   (B) $-\frac{1}{2}$
   (C) $0$
   (D) $\frac{1}{2}$
   (E) $2$

$$x' = \frac{(1-t^2)'(t+3) - (1-t^2)(t+3)'}{(t+3)^2}$$
$$= \frac{-2t(t+3) - (1-t^2)(1)}{(t+3)^2}$$
$$= \frac{-2t^2 - 6t - 1 + t^2}{(t+3)^2}$$
$$x' = \frac{-t^2 - 6t - 1}{(t+3)^2}$$

$$x'(1) = \frac{-(1)^2 - 6(1) - 1}{((1)+3)^2} = \frac{-1-6-1}{(4)^2} = \frac{-8}{16}$$
$$x'(1) = -\frac{1}{2}$$

2. The line $2x - y = 9$ is tangent to the curve $f(x)$ at the point $(4, -1)$. What is the value of $f'(4)$ ?

   (A) $-2$
   (B) $\frac{1}{2}$
   (C) $2$
   (D) $4$
   (E) $9$

$2x - y = 9$ is $f'(x)$ at $(4,-1)$

$-y = -2x + 9$

$y = 2x - 9$

$m = 2$

3. If $f(x) = e^x$, which of the following is equal to $f'(e)$ ?

   (A) $\lim\limits_{h\to 0} \frac{e^{x+h}}{h}$
   (B) $\lim\limits_{h\to 0} \frac{e^{x+h} - e^e}{h}$
   (C) $\lim\limits_{h\to 0} \frac{e^{e+h} - e}{h}$
   (D) $\lim\limits_{h\to 0} \frac{e^{e+h} - 1}{h}$
   (E) $\lim\limits_{h\to 0} \frac{e^{e+h} - e^e}{h}$

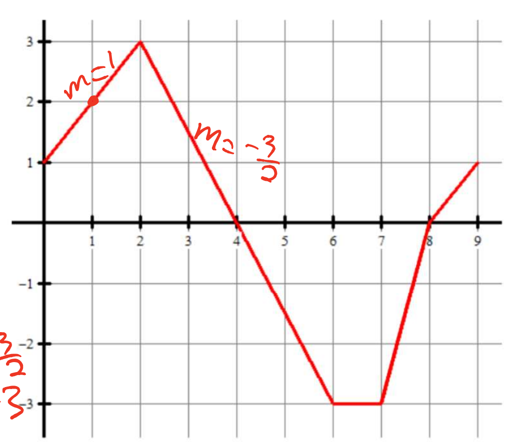
Def'n of f'

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$
$$f'(e) = \lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$$

4. The graph of $f(x)$ is shown below. What is the value of $f(1) + f'(1) + 2f'(4)$ ?

   (A) $0$
   (B) $1$
   (C) $2$
   (D) $3$
   (E) $4$

$f(1) + f'(1) + 2f'(4)$
$= (2) + (1) + -3$
$= 0$




$f(1) = 2$
$f'(1) = 1$
$f'(4) = -\frac{3}{2}$
$2f'(4) = -3$