4.2 Inverse Derivatives

Calculus Name: ____________________ **Practice**

Find the following.

1. $\frac{d}{dx}\sin^{-1}(5x) = \frac{1}{\sqrt{1-u^2}}u'$

$u=5x$
$u'=5$
$u^2=25x^2$

$= \frac{5}{\sqrt{1-25x^2}}$

2. $\frac{d}{dx}\csc^{-1}(4x^5) = -\frac{1}{|u|\sqrt{u^2-1}}u'$

$u=4x^5$
$u^2=16x^{10}$
$u'=20x^4$

$= \frac{-20x^4}{|4x^5|\sqrt{16x^{10}-1}}$

3. $\frac{d}{dx}\tan^{-1}(2x) = \frac{1}{u^2+1}\cdot u'$

$u=2$
$u'=2$
$u^2=4x^2$

$= \frac{2}{4x^2+1}$

4. $\frac{d}{dx}\frac{\sin x}{x} = \frac{(\sin x)'\cdot x - \sin x\cdot x'}{x^2}$

$= \frac{\cos x\cdot x - \sin x\cdot 1}{x^2}$

$= \frac{x\cos x - \sin x}{x^2}$

5. $\frac{d}{dx}\sec^{-1}(x^3) = \frac{1}{|u|\sqrt{u^2-1}}\cdot u'$

$u=x^3$
$u'=3x^2$
$u^2=x^6$

$= \frac{3x^2}{|x^3|\sqrt{x^6-1}}$

6. $\frac{d}{dx}\csc 6x = -\csc 6x\cdot\cot 6x\cdot(6x)'$

Chain $= -6\csc 6x\cot 6x$

7. $\lim_{x\to 2}\frac{x-2}{x^2+5x-14} = \lim_{x\to 2}\frac{x-2}{(x-2)(x+7)}$

$= \lim_{x\to 2}\frac{1}{x+7} = \frac{1}{(2)+7} = \frac{1}{9}$

8. $\frac{d}{dx}\cos^{-1}(3x^2) = -\frac{1}{\sqrt{1-u^2}}\cdot u'$

$u=3x^2$
$u^2=9x^4$
$u'=6x$

$= \frac{-6x}{\sqrt{1-9x^4}}$

9. Anti-derivative of $f'(x) = \frac{5}{\sqrt{1-25x^2}}$

$u'=5$
$u=5x$
$u^2=25x^2$

$f(x) = \sin^{-1}(5x)$

10. $\frac{d}{dx}\cot^{-1}(-x) = -\frac{1}{u^2+1}u'$

$u=-x$
$u'=-1$
$u^2=x^2$

$= \frac{1}{x^2+1}$

11. Anti-derivative of $f'(x) = -\frac{6x^2}{1+4x^6}$

$u^2=4x^6$
$u=2x^3$
$u'=6x^2$

$f(x) = \cot^{-1}(2x^3)$

12. $\frac{d}{dx}\log_5 4x = \frac{1}{u}\cdot\frac{1}{\ln a}\cdot u'$

$= \frac{1}{4x}\cdot\frac{1}{\ln 5}(4)$

$= \frac{1}{x\ln 5}$

13. $\frac{d}{dx}\cos^{-1}(-7x) = \frac{-1}{\sqrt{1-u^2}}u'$

$u=-7x$
$u'=-7$
$u^2=49x^2$

$= \frac{7}{\sqrt{1-49x^2}}$

14. $\frac{d}{dx}\csc^{-1}(x^7) = \frac{-1}{|u|\sqrt{u^2-1}}\cdot u'$

$u=x^7$
$u'=7x^6$
$u^2=x^{14}$

$= \frac{-7x^6}{|x^7|\sqrt{x^{14}-1}}$

15. $\frac{d}{dx}\cot^{-1}(4x^4) = \frac{-1}{u^2+1}\cdot u'$

$u=4x^4$
$u'=16x^3$
$u^2=16x^8$

$= \frac{-16x^3}{16x^8+1}$

16. $\frac{d}{dx}e^{2x^5} = a^u\cdot\ln a\cdot u'$

$= e^{2x^5}\cdot\ln e\cdot 10x^4$

$= 10x^4\cdot e^{2x^5}$

17. $\frac{d}{dx}\tan^{-1}(\sqrt{x}) = \frac{1}{u^2+1}\cdot u'$

$u=\sqrt{x}$
$u'=\frac{1}{2}x^{-\frac{1}{2}}$
$u^2=x$

$= \frac{1}{x+1}\cdot\frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x}(x+1)}$

18. $\frac{d}{dx}5x\sin^{-1}(2x^2)$

Product Rule!

$u=2x^2$ $u^2=4x^4$
$u'=4x$

$=(5x)'\cdot\sin^{-1}(2x^2) + 5x\cdot[\sin^{-1}(2x^2)]'$

$= 5\cdot\sin^{-1}(2x^2) + 5x\cdot\frac{1}{\sqrt{1-u^2}}\cdot u'$

$= 5\sin^{-1}(2x^2) + 5x\frac{4x}{\sqrt{1-4x^4}}$

$= 5\sin^{-1}(2x^2) + \frac{20x^2}{\sqrt{1-4x^4}}$

19. Anti-derivative of

$f'(x) = \dfrac{\cancel{7}\ 21x^6}{\cancel{|x|}\ |3x^7|\sqrt{9x^{14}-1}}$

$u^2 = 9x^{14}$
$u = 3x^7$
$u' = 21x^6$

$f(x) = \sec^{-1}(3x^7)$

20. $\frac{d}{dx}\tan(e^x) = \sec^2(e^x)\cdot(e^x)'$ Chain

$= e^x \cdot \sec^2(e^x)$

21. $\frac{d}{dx}\sec^{-1}(3\ln x) = \dfrac{1}{|u|\sqrt{u^2-1}}\,u'$

$\boxed{u = 3\ln x \quad u^2 = 9\ln^2 x \quad u' = 3\cdot\frac{1}{x}}$

$= \dfrac{1}{|3\ln x|\sqrt{9\ln^2 x - 1}}\cdot\dfrac{3}{x}$

$= \dfrac{3}{x\cdot|3\ln x|\sqrt{9\cdot\ln^2 x - 1}}$

22. $\frac{d}{dx}\sin^{-1}(\sin x) = \dfrac{1}{\sqrt{1-u^2}}\cdot u'$

$\boxed{u = \sin x \quad u' = \cos x \quad u^2 = \sin^2 x}$

$= \dfrac{\cos x}{\sqrt{1-\sin^2 x}}$

$= \dfrac{\cos x}{\sqrt{\cos^2 x}}$

$= \dfrac{\cos x}{\cos x}$

$= 1$

23. $\frac{d}{dx}\dfrac{15x^3+3x^2+55x}{3x}$

Reduce

$= \frac{d}{dx}\left(5x^2 + x + \frac{55}{3}\right)$

$= 10x + 1$

24. Anti-derivative of

$f'(x) = -\dfrac{8x}{\sqrt{1-16x^4}}$

$u^2 = 16x^4$
$u = 4x^2$
$u' = 8x$

$f(x) = \cos^{-1}(4x^2)$

INVERSE FUNCTIONS:

25. If $f(x) = 3x^2$ and $f^{-1}(27) = 3$, find the derivative of $f^{-1}(x)$ at $x = 27$

1 $[f^{-1}(27)]' = \dfrac{1}{f'[f^{-1}(27)]}$

$= \dfrac{1}{f'(3)}$ 2

$[f^{-1}(27)]' = \dfrac{1}{18}$ 5

3 f^{-1}

$\boxed{f^{-1}(27) = 3}$

4 f'

$\boxed{f' = 6x \quad f'(3) = 6(3) \quad f'(3) = 18}$

26. If $f(x) = \cos 3x$ and $f^{-1}(0) = \frac{\pi}{6}$, find the derivative of $f^{-1}(x)$ at $x = 0$

1 $[f^{-1}(0)]' = \dfrac{1}{f'[f^{-1}(0)]}$

$= \dfrac{1}{f'(\frac{\pi}{6})}$ 4

$[f^{-1}(0)]' = \dfrac{1}{-3}$ 5

2 f^{-1}

$\boxed{f^{-1}(0) = \frac{\pi}{6}}$

3 f'

$f' = -\sin 3x\cdot(3x)'$
$f' = -3\sin 3x$
$f'(\frac{\pi}{6}) = -3\cdot\sin(3\cdot\frac{\pi}{6})$
$= -3\sin\frac{\pi}{2}$
$= -3\cdot 1$
$f'(\frac{\pi}{6}) = -3$

27. If $f(x) = x^2 + x$ and $f^{-1}(2) = -2$, find the derivative of $f^{-1}(x)$ at $x = 2$

1 $\boxed{f'(x) = 2x + 1}$

2 $\frac{d}{dx}(f^{-1}(2)) = \dfrac{1}{f'(f^{-1}(2))}$

$= \dfrac{1}{f'(-2)}$

4 $= \dfrac{1}{-3}$

3 $f'(-2) = 2(-2) + 1$
$= -4 + 1$
$f'(-2) = -3$

28. If $f(x) = 6x - 2$ find $\frac{d}{dx}[f^{-1}(x)]$ at $x = 16$

1 f^{-1}

$x = 6y - 2$
$x + 2 = 6y$
$\frac{1}{6}x + \frac{1}{3} = y$
$f^{-1}(x) = \frac{1}{6}x + \frac{1}{3}$
$f^{-1}(16) = \frac{1}{6}(16) + \frac{1}{3}$
$f^{-1}(16) = \frac{8}{3} + \frac{1}{3}$
$f^{-1}(16) = \frac{9}{3} = 3$

1 $[f^{-1}(16)]' = \dfrac{1}{f'[f^{-1}(16)]}$

$= \dfrac{1}{f'(3)}$ 2

$[f^{-1}(16)]' = \dfrac{1}{6}$ 3

4 $\boxed{f' = 6 \quad f'(3) = 6}$

29. If $f(x) = \frac{\sqrt{x}}{3}$ find $\frac{d}{dx}[f^{-1}(x)]$ at $x = 1$

1 $\frac{d}{dx}[f^{-1}(1)] = \dfrac{1}{f'[f^{-1}(1)]}$

$= \dfrac{1}{f'(9)}$ 3

$\frac{d}{dx}[f^{-1}(1)] = \dfrac{1}{18}$ 5

2 f^{-1}

$x = \frac{\sqrt{y}}{3}$
$3x = \sqrt{y}$
$9x^2 = y$
$f^{-1}(x) = 9x^2$
$f^{-1}(1) = 9(1)^2$
$= 9(1)$
$f^{-1}(1) = 9$

4 f'

$f'(x) = \frac{1}{6\sqrt{x}}$
$f'(9) = \frac{1}{6\sqrt{9}}$
$= \frac{1}{6\cdot 3}$
$f'(9) = \frac{1}{18}$

30. If $f(x) = 3x^3 - 4$ find $\frac{d}{dx}[f^{-1}(x)]$ at $x = 20$

1 $[f^{-1}(20)]' = \dfrac{1}{f'[f^{-1}(20)]}$

$= \dfrac{1}{f'(2)}$ 3

$[f^{-1}(20)]' = \dfrac{1}{36}$ 5

2 f^{-1}

$x = 3y^3 - 4$
$x + 4 = 3y^3$
$\frac{x+4}{3} = y^3$
$\sqrt[3]{\frac{x+4}{3}} = y$
$f^{-1}(20) = \sqrt[3]{\frac{20+4}{3}}$
$= \sqrt[3]{\frac{24}{3}}$
$= \sqrt[3]{8}$
$f^{-1}(20) = 2$

4 f'

$f'(x) = 9x^2$
$f'(2) = 9(2)^2$
$= 9(4)$
$f'(2) = 36$