4.3 L'Hôpital's Rule

Calculus Name: ______________________

Practice

Find the following. Use L'Hôpital's when possible.

1. $\lim_{x\to 1} \frac{x-1}{x^2-3x+2} = \frac{0}{0}$

$\lim_{x\to 1} \frac{1}{2x-3} = \frac{1}{2(1)-3}$

$= \frac{1}{2-3}$

$= \frac{1}{-1}$

$= -1$

2. $\lim_{x\to -5} \frac{x^2-2x-35}{x+5} = \frac{0}{0}$

$\lim_{x\to -5} (2x-2) = 2(-5)-2$

$= -10-2$

$= -12$

3. $\lim_{x\to 0} \frac{4x}{\ln(x+1)} = \frac{0}{0}$

$\lim_{x\to 0} \frac{4}{\frac{1}{x+1}\cdot\frac{1}{\ln e}\cdot(1)} =$

$\lim_{x\to 0} \frac{4}{\frac{1}{x+1}} = \frac{4}{\frac{1}{0+1}} = \frac{4}{\frac{1}{1}} = 4$

4. $\lim_{x\to 0} \frac{x-1}{x^2-3x+2} = \frac{0-1}{(0)^2-3(0)+2}$

$= -\frac{1}{2}$

5. $\lim_{x\to 1} \frac{2(x^2-1)}{\ln x^2} = \frac{0}{0}$

$\lim_{x\to 1} \frac{2x^2-2}{2\ln x} =$

$\lim_{x\to 1} \frac{4x}{2\cdot\frac{1}{x}\cdot\frac{1}{\ln e}\cdot 1} =$

$\lim_{x\to 1} \frac{4x}{\frac{2}{x}} = \frac{4(1)}{\frac{2}{1}} = \frac{4}{2} = 2$

6. $\frac{d}{dx}\frac{6x^2+x}{\sin(x)}$ Quotient Rule!

$= \frac{(6x^2+x)'\sin(x) - (6x^2+x)(\sin x)'}{[\sin x]^2}$

$= \frac{(12x+1)\sin(x) - (6x^2+x)(-\cos x)}{\sin^2 x}$

$= \frac{(12x+1)\sin x + \cos x(6x^2+x)}{\sin^2 x}$

7. $\lim_{x\to 0} \frac{2x^2}{e^x-1-x} = \frac{0}{0}$

$= \lim_{x\to 0} \frac{4x}{e^x\cdot\ln e\cdot 1 - 0 - 1}$

$= \lim_{x\to 0} \frac{4x}{e^x-1}$

$= \lim_{x\to 0} \frac{4}{e^x}$

$= \frac{4}{e^0} = \frac{4}{1} = 4$

8. $\lim_{x\to 0} \frac{2x^2}{1-\cos(4x)} = \frac{0}{0}$

$= \lim_{x\to 0} \frac{4x}{-\sin(4x)(4x)'}$

$= \lim_{x\to 0} \frac{4x}{-4\sin(4x)}$

$= \lim_{x\to 0} \frac{4}{-4(-\cos(4x))(4x)'}$

$= \lim_{x\to 0} \frac{1}{\cos 4x\cdot 4} = \frac{1}{\cos(4\cdot 0)\cdot 4} = \frac{1}{4\cos 0} = \frac{1}{4\cdot 1} = \frac{1}{4}$

9. $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} = \frac{0}{0}$

$= \lim_{x\to 0} \frac{\frac{1}{2}(4+x)^{-\frac{1}{2}}(4+x)'}{1}$

$= \lim_{x\to 0} \frac{1\cdot 1}{2\sqrt{4+x}}$

$= \frac{1}{2\sqrt{4+0}} = \frac{1}{2\sqrt{4}} = \frac{1}{2\cdot 2} = \frac{1}{4}$

10. $\lim_{x\to -3} \frac{x-1}{x^2+7x+10}$

$= \frac{(-3)-1}{(-3)^2+7(-3)+10}$

$= \frac{-4}{9-21+10}$

$= \frac{-4}{-2}$

$= 2$

11. $\frac{d}{dx}\frac{6x^2+x}{x+1}$ Quotient Rule!

$= \frac{(6x^2+x)'(x+1) - (6x^2+x)(x+1)'}{(x+1)^2}$

$= \frac{(12x+1)(x+1) - (6x^2+x)(1)}{(x+1)^2}$

$= \frac{12x^2+13x+1-6x^2-x}{(x+1)^2}$

$= \frac{6x^2+12x+1}{(x+1)^2}$

12. $\lim_{x\to 0} \frac{\cos x-1}{x^2} = \frac{0}{0}$

$= \lim_{x\to 0} \frac{-\sin x}{2x}$

$= \lim_{x\to 0} \frac{-\cos x}{2}$

$= \frac{-\cos(0)}{2}$

$= -\frac{1}{2}$

13. $\lim_{x\to\infty} \frac{e^{2x}}{2x^2} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{e^{2x}\cdot\ln e\cdot 2}{4x} = \lim_{x\to\infty} \frac{2e^{2x}}{4x}$

$= \lim_{x\to\infty} \frac{2e^{2x}\cdot\ln e\cdot 2}{4} = \lim_{x\to\infty} \frac{4e^{2x}}{4}$

$= \lim_{x\to\infty} e^{2x} = \infty$, dne

14. $\lim_{x\to\infty} \frac{\ln x^2}{\ln(x+4)^3} = \lim_{x\to\infty} \frac{2\ln x}{3\ln(x+4)}$

$= \lim_{x\to\infty} \frac{2\cdot\frac{1}{x}\cdot\frac{1}{\ln e}\cdot 1}{3\cdot\frac{1}{x+4}\cdot\frac{1}{\ln e}\cdot(1)}$

$= \lim_{x\to\infty} \frac{\frac{2}{x}}{\frac{3}{x+4}} = \lim_{x\to\infty} \frac{2}{x}\cdot\frac{x+4}{3}$

$= \lim_{x\to\infty} \frac{2x+8}{3x}$ } growing at same rate

$= \frac{2}{3}$

15. $\lim_{x\to -2} \frac{x+2}{x^2+2x-3} = \frac{(-2)+2}{(-2)^2+2(-2)-3}$

$= \frac{0}{4-4-3}$

$= \frac{0}{-3}$

$= 0$

16. $\frac{d}{dx}\frac{e^x}{\cos(2x)} = \frac{(e^x)'\cos(2x) - e^x\cdot[\cos(2x)]'}{[\cos(2x)]^2}$

$= \frac{e^x\cdot\cos(2x) - e^x(-\sin(2x))\cdot(2x)'}{\cos^2(2x)}$

$= \frac{e^x\cos(2x) + 2e^x\sin(2x)}{\cos^2(2x)}$

17. $\lim_{x\to 0}\frac{\sqrt{1+x}-1-\frac{x}{2}}{x^2} = \frac{0}{0}$

$= \lim_{x\to 0}\frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}(1+x)' - \frac{1}{2}}{2x}$

$= \lim_{x\to 0}\frac{\frac{1}{2}(1+x)^{-\frac{1}{2}} - \frac{1}{2}}{2x} = \frac{0}{0}$

$= \lim_{x\to 0}\frac{-\frac{1}{4}(1+x)^{-3/2}(1+x)'}{2}$

$= \lim_{x\to 0}\frac{-\frac{1}{4}(1+x)^{-3/2}}{2} = \frac{-\frac{1}{4}(1+0)^{-3/2}}{2} = \frac{-\frac{1}{4}(1)^{-3/2}}{2} = \frac{-\frac{1}{4}(1)}{2} = \frac{-1}{8}$

18. $\lim_{x\to 10}\frac{5-\sqrt{x+15}}{x-10} = \frac{0}{0}$

$= \lim_{x\to 10}\frac{-\frac{1}{2}(x+15)^{-\frac{1}{2}}(x+15)'}{1}$

$= \lim_{x\to 10} -\frac{1}{2}(x+15)^{-\frac{1}{2}}$

$= -\frac{1}{2}(10+15)^{-\frac{1}{2}}$

$= -10\,(25)^{-\frac{1}{2}}$

$= -10\left(\frac{1}{5}\right)$

$= -2$

19. $\lim_{x\to -5}\frac{x^2-2x-15}{x+5} = \lim_{x\to -5}\frac{(x+5)(x-3)}{x+5}$

$= \lim_{x\to -5}(x-3)$

$= -5-3$

$= -8$

4.3 L'Hôpital's Rule

Test Prep

1. $\lim_{x\to 2}\frac{e^{2x}-e^4}{x-2} \; \begin{matrix}=0\\=0\end{matrix} = \lim_{x\to 2}\frac{2e^{2x}-0}{1} = \lim_{x\to 2} 2e^{2x} = 2e^{2(2)} = 2e^4$

(A) e (B) $2e$ (C) $2e^2$ (D) e^4 (E) $2e^4$

2. If $f(x) = x\sqrt{4x-1}$, then $f'(x)$ is

$f'(x) = x'(4x-1)^{\frac{1}{2}} + x\cdot[(4x-1)^{\frac{1}{2}}]'$

$= 1\cdot(4x-1)^{\frac{1}{2}} + x\cdot\frac{1}{2}(4x-1)^{-\frac{1}{2}}(4x-1)'$

$= (4x-1)^{\frac{1}{2}} + \frac{1}{2}x\cdot(4x-1)^{-\frac{1}{2}}(4)$

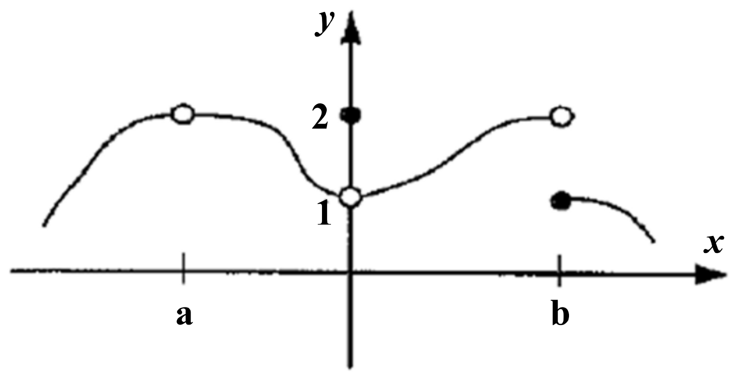
$= (4x-1)^{-\frac{1}{2}}\left[(4x-1) + \frac{1}{2}x\cdot 4\right]$

$= (4x-2)^{-\frac{1}{2}}[4x-1+2x]$

$= \frac{6x-1}{\sqrt{4x-1}}$

(A) $\frac{6x-1}{\sqrt{4x-1}}$ (B) $\frac{2x}{\sqrt{4x-1}}$ (C) $\frac{1}{\sqrt{4x-1}}$

(D) $\frac{-6x+1}{\sqrt{4x-1}}$ (E) $\frac{9x-2}{2\sqrt{4x-1}}$



3. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $f(a)$ exists False

(B) $\lim_{x\to a} f(x) = 2$ True

(C) $\lim_{x\to b} f(x) = 1$ False

(D) $\lim_{x\to b^-} f(x) = \lim_{x\to b^+} f(x)$ False

(E) f is continuous at $x = 0$ False

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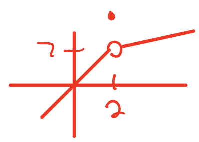
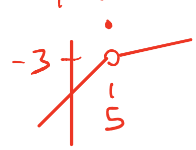
4. Let f be a function defined for all real numbers. Which of the following statements about f must be true?

(A) If $\lim_{x\to 2} f(x) = 7$, then $f(2) = 7$.



False

(B) If $\lim_{x\to 5} f(x) = -3$, then -3 is in the range of f.



False

(C) If $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$, then $f(1)$ exists. False

(D) If $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$, then $\lim_{x\to 3} f(x)$ does not exist.

(E) If $\lim_{x\to 4} f(x)$ does not exist, then $f(4)$ does not exist.

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5. The slope to the tangent line to the graph of $y = \tan(2x)$ at $x = \frac{\pi}{8}$ is

$y' = \sec^2(2x)\cdot(2x)'$

$y' = 2\sec^2(2x)$

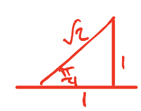
$y'\left(\frac{\pi}{8}\right) = 2\sec^2\left[2\left(\frac{\pi}{8}\right)\right]$

$= 2\sec^2\left(\frac{\pi}{4}\right)$

$= 2\left(\sqrt{2}\right)^2$

$= 2\cdot 2$

$= 4$



(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) 2

(D) $2\sqrt{2}$

(E) 4

6. An object moves along the y-axis with the coordinate position $y(t)$ and velocity $v(t) = \sqrt{t} - \cos(e^t)$ for $t \geq 0$. At time $t = 1$, the object is

~~(A) moving downward with negative acceleration~~
~~(B) moving upward with negative acceleration~~
(C) moving downward with positive acceleration
(D) moving upward with positive acceleration
(E) at rest

$a(t) = \frac{1}{2}t^{-\frac{1}{2}} + \sin(e^t)(e^t)'$
$a(1) = \frac{1}{2\sqrt{t}} + e^t \sin(e^t)$
$= \frac{1}{2\sqrt{1}} + e^1 \sin e^1$
$= \frac{1}{2} + e \sin e$
$\approx \frac{1}{2} + 1.116$
$\approx$ Positive

$v(1) = \sqrt{1} - \cos(e^1)$
$= 1 - \cos(e)$
$\approx 1 - (-.911)$
$v(1) \approx 1.911$

FREE RESPONSE **Your score: ________ out of 6**

Use the space below the problem to show work and solutions. Score your answers when completed.

x	-3	-2	-1	0	1	2	3
$f(x)$	5	3	2	-1	1	-4	-7

1. The table above shows selected values for a twice-differentiable function f.

   (a) Show that the average rate of change of f over the interval $-3 \leq x \leq 0$ is the same as the average rate of change of f over the interval $0 \leq x \leq 3$.

   (b) Explain why there must be at least one value c such that $0 < c < 2$ and $f'(c) = 0$.

   (c) Must there be at least one value d such that $-3 < d < 3$ and $f''(d) = 0$. Explain why or why not.

Free Response Scoring Guide

Use this only AFTER you have attempted the problem on your own.

ARC$(-3,0)$ =

ARC$(0,3)$ =

Solutions	Points
(a) $\frac{f(0)-f(-3)}{3} = \frac{-1-5}{3} = \frac{-6}{3} = -2$ $\frac{f(3)-f(0)}{3} = \frac{-7-(-1)}{3} = \frac{-6}{3} = -2$	2: average rate of change for each interval
(b) f has an average increase over the interval $0 \leq x \leq 1$ and an average decrease over the interval $1 \leq x \leq 2$. This means somewhere over the interval $0 \leq x \leq 2$ there must be a value c such that $f(c)$ is a maximum. This maximum would yield $f'(c) = 0$ because f is differentiable.	1: Increasing and then decreasing 1: differentiable maximum means $f'(c) = 0$
(c) From part (a) and the MVT, there is a value r between -3 and 0 such that $f'(r) = -2$ and there is a value s between 0 and 3 such that $f'(s) = -2$. Using the MVT again, there must be a value d between r and s such that $f''(d) = 0$. (One could also argue that f decreases, increases, and decreases on the interval, therefore there are two points where $f'(x) = 0$ and between them there must be at least one point where $f''(d) = 0$.)	1: Use the MVT to state the first derivative equals -2 at some point. 1: Use the MVT again to state the second derivative equals 0 at some point.