4.2 Inverse Derivatives

Test Prep

1. Compute the derivative of $f(x) = \ln x - \sin x + \arctan x + 2^x, x > 0$.

$f'(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + 2^x \cdot \frac{1}{\ln 2}$

(A) $f(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + x2^x$

(B) $f(x) = \frac{1}{x} - \cos x + \frac{1}{1-x^2} + x2^x$

(C) $f(x) = \frac{1}{x} + \cos x + \frac{1}{1-x^2} + (\ln 2)2^x$

(D) $f(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + (\ln 2)2^x$

(E) $f(x) = \frac{1}{x} + \cos x + \frac{1}{1+x^2} + (\ln 2)2^x$

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2. What is an equation for the line tangent to $y = \tan^{-1} x$ at $x = \sqrt{3}$?

Point $(\sqrt{3}, \frac{\pi}{3})$

$y = \tan^{-1}(\sqrt{3})$

$y = \frac{\pi}{3}$

Slope

$y' = \frac{1}{x^2+1}$

$y'(\sqrt{3}) = \frac{1}{(\sqrt{3})^2+1}$

$= \frac{1}{3+1}$

$y'(\sqrt{3}) = \frac{1}{4}$

(A) $y - \frac{\pi}{3} = -\frac{1}{2}(x - \sqrt{3})$

(B) ~~$y - \frac{\pi}{6} = -\frac{1}{4}(x - \sqrt{3})$~~

(C) $y - \frac{\pi}{3} = -\frac{1}{4}(x - \sqrt{3})$

(D) ~~$y - \frac{\pi}{6} = \frac{3}{4}(x - \sqrt{3})$~~

(E) $y - \frac{\pi}{3} = \frac{1}{4}(x - \sqrt{3})$

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3. $\lim_{x \to -\infty} \frac{2x+3}{\sqrt{x^2+x+1}}$ is

Same growth rate but negative

Same growth rate positive

$= \frac{-2}{1}$

(A) -2

(B) -1

(C) 0

(D) 2

(E) nonexistent

$x = 3y^2 - y$

$f' = 6x - 1$

$f'(2) = 6(2) - 1 = 12 - 1 = 11$

4. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be

$g'(10) = \frac{1}{f'[g(10)]} = \frac{1}{f'(2)} = \frac{1}{11}$

$10 = 3x^2 - x$

$0 = 3x^2 - x - 10$

$0 = (3x^2 - 6x) + (5x - 10)$

$0 = 3x(x-2) + 5(x-2)$

$0 = (x-2)(3x+5)$

$x = 2, -5/3$

(A) 59 (B) $\frac{1}{59}$ (C) $\frac{1}{10}$ (D) 11 (E) $\frac{1}{11}$

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5. If $f(x) = x^{\frac{5}{2}}$, then $f'(4)$?

$f'(x) = \frac{5}{2} x^{3/2}$

$f'(4) = \frac{5}{2}(\sqrt{4})^3 = \frac{5}{2}(2)^3 = \frac{5}{2}(8) = 5 \cdot 4 = 20$

(A) -10 (B) 24 (C) 5 (D) 10 (E) 20

FREE RESPONSE **Your score: ________ out of 3**

2007 Form A AB3

Use the space below the problem to show work and solutions. Score your answers when completed.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

1. The functions f and g are differentiable for all real numbers. The table above gives values of the functions and their first derivatives at selected values of x.

   (a) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

Free Response Scoring Guide

Use this only AFTER you have attempted the problem on your own.

Solutions	Points
$g(1) = 2$, so $g^{-1}(2) = 1$. $(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5}$ An equation of the tangent line is $y - 1 = \frac{1}{5}(x-2)$.	$3: \begin{cases} 1: g^{-1}(2) \\ 1: (g^{-1})'(2) \\ 1: \text{tangent line equation} \end{cases}$