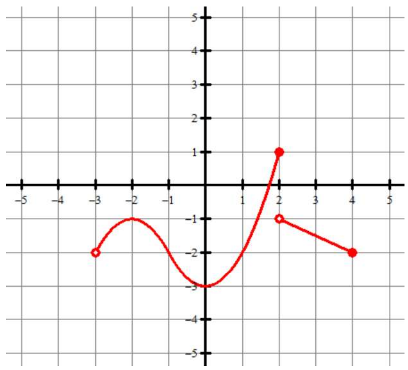
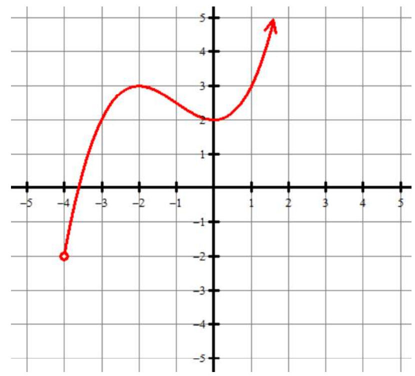
Find the extreme values and where they occur.

1.



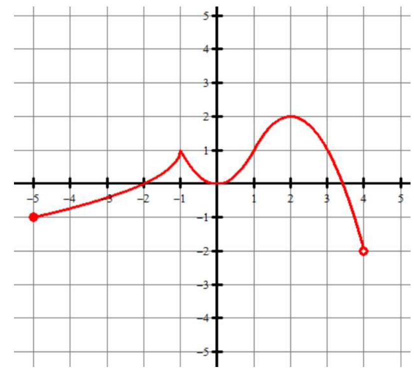
Rel MAX of -1 when x = -2
Rel MIN of -2 when x = 4
ABS MAX of 1 when x = 2
ABS MIN of -3 when x = 0

2.



Rel MAX of 3 when x = -2
Rel MIN of 2 when x = 0
~~ABS MAX of when x =~~
~~ABS MIN of when x =~~

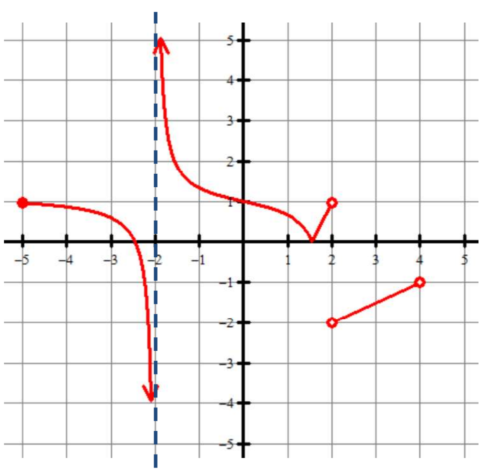
3.



Rel MAX of 1 when x = -1
Rel MIN of -1, 0 when x = -5, 0
ABS MAX of 2 when x = 2
~~ABS MIN of when x =~~

Use the graph of $f(x)$ to answer the following.

4.



Domain: $[-5,-2)\cup(-2,2)\cup(2,4)$

$\lim_{x\to 2^+} f(x) = -2$

$\lim_{x\to -2} f(x) = 1$

$\lim_{x\to 0} f(x) = 1$

$f(3) = -1.5$

$f'(3) = \frac{1}{2}$ (slope of segment)

Average rate of change over $[-5,-3]$

Absolute max: DNE

Absolute min: DNE

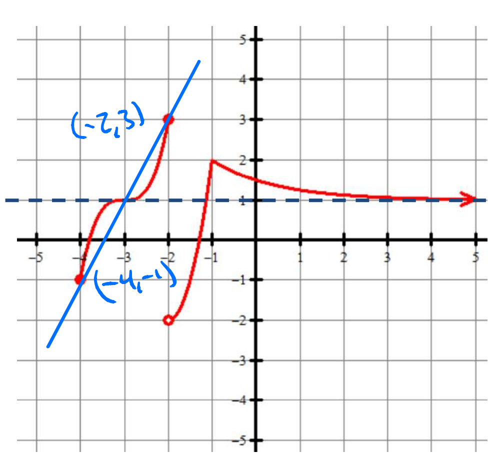
Local max: $(-5, 1)$

Local min: $(1.5, 0)$

Interval(s) where $f(x)$ increasing
$\left(\frac{3}{2}, 2\right)\cup(2, 4)$

Interval(s) where $f(x)$ decreasing
$(-5,-2)\cup\left(-2, \frac{3}{2}\right)$

5.



Domain: $[-4, \infty)$

$\lim_{x\to -2^+} f(x) = -2$

$\lim_{x\to -2} f(x) = \text{DNE}$

$\lim_{x\to \infty} f(x) = 1$

$f(-3) = 1$

$f'(-1) = \text{DNE}$

Average rate of change over $[-4,-2]$

$$ARC = \frac{f(b)-f(a)}{b-a} = \frac{(3)-(-1)}{(-2)-(-4)} = \frac{4}{2} = 2$$

Global max: $(-2, 3)$

Global min: DNE

Relative max: $(-1, 2)$

Relative min: $(-4, -1)$

Interval(s) where $f(x)$ increasing
$(-4,-2)\cup(-2,-1)$

Interval(s) where $f(x)$ decreasing
$(-1, \infty)$

values from f'

Find the critical ~~points.~~

6. $f(x) = 4x^3 - 9x^2 - 12x + 3$

CV

$f' = 12x^2 - 18x - 12$

$0 = 6(2x^2 - 3x - 2)$

$0 = 6[(2x^2 - 4x) + (x - 2)]$

$0 = 6[2x(x-2) + 1(x-2)]$

$0 = 6(x-2)(2x+1)$

$0 \neq 6$; $0 = x - 2$, $2 = x$; $2x + 1 = 0$, $2x = -1$, $x = -1/2$

CV: $x = -\frac{1}{2}, 2$

7. $g(t) = \frac{2}{t^2-4} = 2(t^2-4)^{-1}$

CV

$g'(t) = -2(t^2-4)^{-2}(t^2-4)'$

$0 = \frac{-2}{(t^2-4)^2}(2t)$

$0 = \frac{-4t}{(t^2-4)^2}$

ZON: $-4t = 0$, $t = 0$

ZOD: $(t^2-4)^2 = 0$, $t^2 - 4 = 0$, $t^2 = 4$, $t = \pm 2$

CV: $x = -2, 0, 2$

8. $h(x) = \sqrt[3]{x-2}$

CV

$h'(x) = \frac{1}{3}(x-2)^{-2/3}(x-2)'$

$h'(x) = \frac{1}{3\sqrt[3]{(x-2)^2}}$

ZON: $1 \neq 0$

ZOD: $3\sqrt[3]{(x-2)^2} = 0$, $\sqrt[3]{(x-2)^2} = 0$, $x - 2 = 0$, $x = 2$

CV: $x = 2$

9. $f(x) = (\ln x)^2$

CV

$f'(x) = 2(\ln x) \cdot \frac{1}{x}$

$0 = \frac{2\ln x}{x}$

ZON: $2\ln x = 0$, $\ln x = 0$, $e^0 = x$, $1 = x$

ZOD: $x = 0$

CV: $x = 0, 1$

10. $h(x) = 2\sin\left(\frac{x}{2}\right)$

where $-2\pi \le x \le 2\pi$

CV

$h'(x) = 2\cos\left(\frac{x}{2}\right) \cdot \left(\frac{x}{2}\right)'$

$h'(x) = \cos\left(\frac{1}{2}x\right)$

$0 = \cos\left(\frac{1}{2}x\right)$

$\cos\frac{\pi}{2} = 0$, $\frac{\pi}{2}$; $\frac{3\pi}{2}$, $\cos\frac{3\pi}{2} = 0$

So, $\frac{1}{2}x = \frac{\pi}{2}$, $x = \pi$; $\frac{1}{2}x = \frac{3\pi}{2}$, $x = 3\pi$ (Not in Domain, but $-\pi$ is)

CV: $x = -\pi, \pi$

11. $g(x) = e^x - x$

CV

$g'(x) = e^x - 1$

$0 = e^x - 1$

$1 = e^x$

$0 = x$

CV: $x = 0$

Find the absolute maximum and minimum values of the function on the given interval.

y-values.

12. $f(x) = 1 + (x+1)^2, \quad [-2, 5]$

CV

$f'(x) = 2(x+1)' \cdot (x+1)'$

$0 = 2(x+1)$

$0 \neq 2$; $0 = x + 1$, $-1 = x$

CV: $x = -1$

EV: $x = -2, 5$

CP & EP

$f(-2) = 2$

$f(-1) = 1$ ABS MIN

$f(5) = 37$ ABS MAX

13. $f(x) = 2x^3 + 3x^2 + 4 \quad [-2, 1]$

CV

$f'(x) = 6x^2 + 6x$

$0 = 6x(x+1)$

$0 = 6x$, $0 = x$; $0 = x + 1$, $-1 = x$

CV: $x = -1, 0$

EV: $x = -2, 1$

CP & EP

$f(-2) = 0$ ABS MIN

$f(-1) = 5$

$f(0) = 4$

$f(1) = 9$ ABS MAX

14. $f(x) = x^3 - 12x, \quad [0, 3)$

CV

$f'(x) = 3x^2 - 12$

$0 = 3(x^2 - 4)$

$0 = 3(x-2)(x+2)$

$0 \neq 3$; $0 = x - 2$, $2 = x$; $0 = x + 2$, $-2 = x$, $-2 \notin [0, 3)$

CV: $x = 2$

EV: $x = 0$

CP

$f(0) = 0$ ABS MAX

$f(2) = -16$ ABS MIN

15. $h(x) = 3x^{\frac{2}{3}} - 2x, \quad [-1, 1]$

CV

$h'(x) = 2x^{-\frac{1}{3}} - 2$

$0 = \frac{2}{\sqrt[3]{x}} - 2 \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}}$

$0 = \frac{2 - 2\sqrt[3]{x}}{\sqrt[3]{x}}$

ZON: $2 - 2\sqrt[3]{x} = 0$, $2 = 2\sqrt[3]{x}$, $1 = \sqrt[3]{x}$, $1 = x$

ZOD: $\sqrt[3]{x} = 0$, $x = 0$

CV: $x = 0, 1$

EV: $x = -1, 1$

CP & EP

$f(-1) = 5$ ABS MAX

$f(0) = 0$ ABS MIN

$f(1) = 1$

Find the absolute maximum and minimum values of the function on the given interval.

16. $g(x) = x^2 + \frac{2}{x}$, $\left(\frac{1}{2}, 2\right]$

CV

$g'(x) = 2x - 2x^{-2}$

$g'(x) = \frac{x^2 \cdot 2x}{x^2} - \frac{2}{x^2}$

$g'(x) = \frac{2x^3 - 2}{x^2}$

ZON: $2x^3 - 2 = 0$, $2x^3 = 2$, $x^3 = 1$, $x = 1$

ZOD: $x^2 = 0$, $x = 0$, $0 \notin (\frac{1}{2}, 2]$

CV: $x = 1$

EV: $x = 2$

CP & EP

$g(1) = 3$ CP

$g(2) = 5$ EP

ABS MAX value = 5

ABS MIN value = 3

17. $f(x) = \frac{x}{x^2+1}$, $[-2, 2]$

CV

$f' = \frac{x'(x^2+1) - x \cdot (x^2+1)'}{(x^2+1)^2}$

$0 = \frac{1 \cdot (x^2+1) - x(2x)}{(x^2+1)^2}$

$0 = \frac{x^2 + 1 - 2x^2}{(x^2+1)^2}$

$0 = \frac{1 - x^2}{(x^2+1)^2}$

$0 = \frac{(1-x)(1+x)}{(x^2+1)^2}$

ZON: $0 = 1 - x$, $x = 1$; $0 = 1 + x$, $x = -1$

ZOD: $(x^2+1)^2 = 0$, $x^2 + 1 = 0$, $x^2 = -1$, $x = \pm i$

CV: $x = -1, 1$

EV: $x = -2, 2$

CP & EP

$f(-2) = -\frac{2}{5}$ ABS MIN EP

$f(-1) = -\frac{1}{2}$ CP

$f(1) = \frac{1}{2}$ CP

$f(2) = \frac{2}{5}$ ABS MAX EP

18. $f(x) = \sin\left(x + \frac{\pi}{4}\right)$, $\left[0, \frac{7\pi}{4}\right]$

CV

$f'(x) = \cos\left(x + \frac{\pi}{4}\right) \cdot \left(x + \frac{\pi}{4}\right)'$

$0 = \cos\left(x + \frac{\pi}{4}\right)$

$\cos\frac{\pi}{2} = 0$

$\cos\frac{3\pi}{2} = 0$

$x + \frac{\pi}{4} = \frac{\pi}{2}$, $4x + \pi = 2\pi$, $4x = \pi$, $x = \frac{\pi}{4}$

$x + \frac{\pi}{4} = \frac{3\pi}{2}$, $4x + \pi = 6\pi$, $4x = 5\pi$, $x = \frac{5\pi}{4}$

CV: $x = \frac{\pi}{4}, \frac{5\pi}{4}$

EV: $x = 0, \frac{7\pi}{4}$

CP & EP

EP $f(0) = \frac{\sqrt{2}}{2}$

CP $f(\frac{\pi}{4}) = 1$ ABS MAX

CP $f(\frac{5\pi}{4}) = -1$ ABS MIN

EP $f(\frac{7\pi}{4}) = 0$

19. $g(x) = xe^{2x}$, $[-1, 1]$

CV

$g'(x) = x' \cdot e^{2x} + x \cdot (e^{2x})'$

$0 = 1 \cdot e^{2x} + x \cdot 2 \cdot e^{2x}$

$0 = e^{2x} + 2xe^{2x}$

$0 = e^{2x}(1 + 2x)$

$0 = e^{2x}$ DNE

$0 = 1 + 2x$, $-1 = 2x$, $-\frac{1}{2} = x$

CV: $x = -\frac{1}{2}$

EV: $x = -1, 1$

CP & EP

EP $f(-1) = -e^{-2}$

CP $f(-\frac{1}{2}) = -\frac{1}{2}e^{-1}$ ABS MIN

EP $f(1) = e^2$ ABS MAX

MULTIPLE CHOICE

1. If f is a continuous, decreasing function on [0,10] with a critical point at $(4, 2)$, which of the following statements must be false?

   (A) $f(10)$ is an absolute minimum of f on [0,10].

   (B) $f(4)$ is neither a relative maximum nor a relative minimum.

   (C) $f'(4)$ does not exist

   (D) $f'(4) = 0$

   (E) $f'(4) < 0$

Questions 2 and 3 refer to the graph shown on the right.

2. Which of the following statements is false?

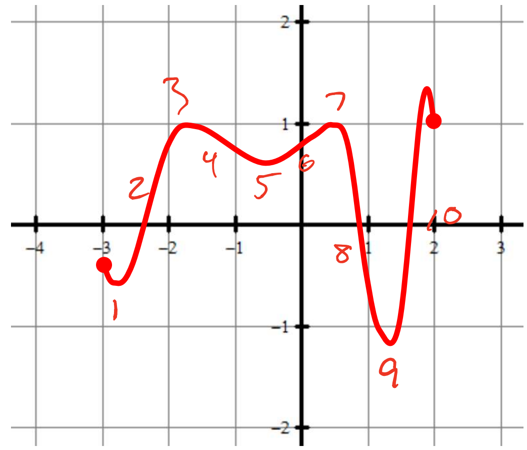
   (A) $F(-3) + F(2) > 0$

   (B) $F(-1) + F'^{(-1)} > 0$

   (C) $F'(-1) \cdot F''(-1) < 0$

   (D) $F(1) \cdot F'(1) < 0$

   (E) $F(0) \cdot F'(0) > 0$



The graph of F

3. The function F has exactly this many critical numbers.

   (A) 4
   (B) 5
   (C) 6
   (D) 7
   (E) 8
   (F) 10

4. Let $x(t) = t^{\frac{2}{3}}$ give the distance of a moving particle from its starting point as a function of time t. For what value of t is the instantaneous velocity of the particle equal to its average velocity over the interval [0,8] ?

   (A) $\frac{8}{27}$

   (B) $\frac{27}{64}$

   (C) $\frac{64}{27}$

   (D) $\frac{27}{8}$

   (E) $\frac{64}{9}$

MVT
Set ARC= f'(c)

5. What is the range of the function $f(x) = \frac{\ln x}{x}$ on the closed interval $[1, e^2]$?

   cv of MIN　cv of MAX

   (A) $f(1) \leq f(x) \leq f(e)$

   (B) $f(1) \leq f(x) \leq f(e^2)$

   (C) $f(2) \leq f(x) \leq f(e)$

   (D) $f(e) \leq f(x) \leq f(e^2)$

   (E) None of these

FIND MAX & MINS

 **You will need a graphing calculator for #6** 

6. Find the value of c that satisfies the Mean Value Theorem for $f(x) = x \sin x$ on [1,4].

   (A) 1.239
   (B) 1.290
   (C) 2.029
   (D) 2.463
   (E) 3.027

MVT
ARC= f'(c)