

5.2 First Derivative Test

PRACTICE

Complete the sign chart and locate all extrema.

1. Given $f(x)$ is continuous and differentiable.

Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 3)$	$(3, \infty)$
Test Value	$x = -4$	$x = -1$	$x = 1$	$x = 4$
$f'(x)$	$f'(-4) = 4$	$f'(-1) = -3$	$f'(1) = -7$	$f'(4) = \frac{1}{2}$
Conclusion				

MAX @ $x = -2$
MIN @ $x = 3$

$f'(-2) = 0$ $f'(3) = 0$

Use a sign chart to find the intervals where the function is increasing or decreasing and relative extrema.

2. $f(x) = x^3 - 12x + 1$
CV

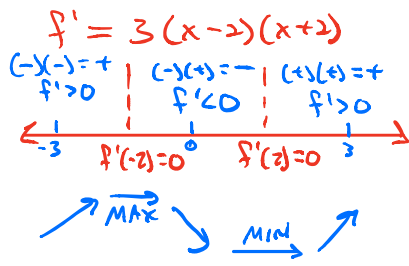
$$f'(x) = 3x^2 - 12$$

$$0 = 3(x^2 - 4)$$

$$0 = 3(x-2)(x+2)$$

$$0 \neq 3 \left\{ \begin{array}{l} 0 = x-2 \\ 2 = x \end{array} \right. \left\{ \begin{array}{l} 0 = x+2 \\ -2 = x \end{array} \right.$$

CV: $x = -2, 2$



- The function is increasing from $(-\infty, -2)$ and $(2, \infty)$.
- The function is decreasing from $(-2, 2)$.
- There is a maximum at $x = -2$ because $f' > 0$ on the left and $f' < 0$ on the right.
- There is a minimum at $x = 2$ because $f' < 0$ on the left and $f' > 0$ on the right.

3. $g(x) = x^2(x-3)$ pfff
 $g(x) = x^3 - 3x^2$

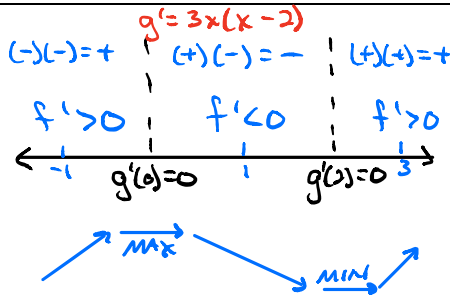
CV

$$g'(x) = 3x^2 - 6x$$

$$0 = 3x(x-2)$$

$$0 = 3x \left\{ \begin{array}{l} 0 = x-2 \\ 2 = x \end{array} \right.$$

CV: $x = 0, 2$



- The function is increasing from $(-\infty, 0)$ and $(2, \infty)$.
- The function is decreasing from $(0, 2)$.
- There is a maximum at $x = 0$ because $f' > 0$ on the left and $f' < 0$ on the right.
- There is a minimum at $x = 2$ because $f' < 0$ on the left and $f' > 0$ on the right.

4. $f(x) = (x^2 - 1)^{\frac{2}{3}}$ Chain Rule

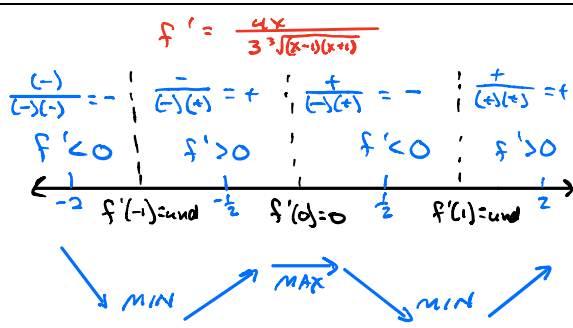
$$f'(x) = \frac{2}{3}(x^2 - 1)^{-\frac{1}{3}}(2x)$$

$$0 = \frac{2}{3 \cdot 2} \frac{2x}{x^2 - 1} \cdot (2x)$$

$$0 = \frac{4x}{3^2 \sqrt{x^2 - 1}}$$

ZON: $x = 0$
ZOD: $\frac{2}{3} \sqrt{x^2 - 1} = 0 \Rightarrow x = \pm 1$

CV: $x = -1, 0, 1$



- The function is increasing from $(-1, 0)$ and $(1, \infty)$.
- The function is decreasing from $(-\infty, -1)$ and $(0, 1)$.
- There is a maximum at $x = 0$ because $f' > 0$ on the left and $f' < 0$ on the right.
- There is a minimum at $x = -1$ and $x = 1$ because $f' < 0$ on the left and $f' > 0$ on the right.

5. $g(t) = 12(1 + \cos t)$ on the interval $[0, 2\pi]$

A particle moves along the x-axis with the position function given below. Find the velocity and use a sign chart to describe the motion of the particle.

6. $h(x) = -x^5 + \frac{5}{2}x^4 + \frac{40}{3}x^3 + 5$ $v = -5x^2(x-4)(x+2)$

CV

$v(x) = -5x^4 + 10x^3 + 40x^2$
 $0 = -5x^2(x^2 - 2x - 8)$
 $0 = -5x^2(x-4)(x+2)$
 $0 = -5x^2 \begin{cases} 0 = x-4 \\ 0 = x^2 - 2x - 8 \end{cases} \begin{cases} 0 = x+2 \\ 4 = x \end{cases} \begin{cases} 0 = x+2 \\ -2 = x \end{cases}$
 $0 = x$
 CV: $x = -2, 0, 4$

$(-)(+)(-)(-) \quad | \quad (-)(+)(-)(-)$ $(-)(+)(-)(+)$ $(-)(+)(+)(+)$
 $v = - \quad | \quad v = + \quad | \quad v = + \quad | \quad v = -$
 $\leftarrow -3 \quad v(-2)=0 \quad -1 \quad v(0)=0 \quad 1 \quad v(4)=0 \quad 5 \rightarrow$
 moving left moving right moving right moving left

The particle is not moving at $x = -2, 0, 4$
 The particle is moving left from $(-\infty, -2)$ and $(4, \infty)$
 The particle is moving right from $(-2, 0)$ and $(0, 4)$

7. $g(x) = e^{\cos x}$ on the interval $[0, 2\pi]$

Given the graph of $f'(x)$, find the critical points and locate all relative extrema.

