



**Two Lies and a Truth Activity: Topic 1.6**

For each of the following limit problems, three solutions are provided. Two of the solutions show an incorrect answer where one solution shows a correct answer. Circle **True** if the solution is correct, or true, or circle **Lie** if the solution is incorrect, and thus a lie. For each solution this is a “lie,” circle where error occurs.

<p><b>Problem 1</b></p> $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + 7x - 8}$	$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + 7x - 8}$ $= \lim_{x \rightarrow 1} \frac{(x-1)(x-1)(x-1)}{(x-1)(x+8)}$ $= \lim_{x \rightarrow 1} \frac{(x-1)^2}{x+8}$ $= -\frac{0^2}{9}$ $= 0$ <p>True <span style="border: 1px solid red; padding: 2px;">Lie</span></p> <p><b>The factoring is incorrect for the difference of two cubes.</b></p>	$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + 7x - 8}$ $= \lim_{x \rightarrow 1} \frac{(x-1)(x^2 + x + 1)}{(x-1)(x+8)}$ $= \lim_{x \rightarrow 1} \frac{x^2 + x + 1}{x+8}$ $= \frac{3}{9} \text{ or } \frac{1}{3}$ <p><span style="border: 1px solid red; padding: 2px;">True</span> Lie</p>	$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 + 7x - 8}$ $= \lim_{x \rightarrow 1} \frac{(x-1)(x^2 - x + 1)}{(x-1)(x+8)}$ $= \lim_{x \rightarrow 1} \frac{x^2 - x + 1}{x+8}$ $= \frac{1}{9}$ <p>True <span style="border: 1px solid red; padding: 2px;">Lie</span></p> <p><b>The factoring is incorrect for the difference of two cubes. The circled sign should be a “plus.”</b></p>
<p><b>Problem 2</b></p> $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x^2}$	$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x^2} \cdot \frac{\sqrt{x^2 + 9} - 3}{\sqrt{x^2 + 9} - 3}$ $= \lim_{x \rightarrow 0} \frac{x^2 + 9 + 9}{x^2 (\sqrt{x^2 + 9} - 3)}$ $= \lim_{x \rightarrow 0} \frac{x^2 + 18}{x^2 (\sqrt{x^2 + 9} - 3)}$ $= \lim_{x \rightarrow 0} \frac{18}{\sqrt{x^2 + 9} - 3}$ $= \infty$ <p>True <span style="border: 1px solid red; padding: 2px;">Lie</span></p> <p><b>The sign of the conjugate is incorrect. It should be a “plus.”</b></p>	$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x^2} \cdot \frac{\sqrt{x^2 + 9} + 3}{\sqrt{x^2 + 9} + 3}$ $= \lim_{x \rightarrow 0} \frac{x^2 + 9 - 9}{x^2 (\sqrt{x^2 + 9} + 3)}$ $= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x^2 + 9} + 3}$ $= \frac{1}{\sqrt{12}}$ <p>True <span style="border: 1px solid red; padding: 2px;">Lie</span></p> <p><b>When evaluating the limit, the 3 was misinterpreted as being under the square root.</b></p>	$\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 9} - 3}{x^2} \cdot \frac{\sqrt{x^2 + 9} + 3}{\sqrt{x^2 + 9} + 3}$ $= \lim_{x \rightarrow 0} \frac{x^2 + 9 - 9}{x^2 (\sqrt{x^2 + 9} + 3)}$ $= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x^2 + 9} + 3}$ $= \frac{1}{6}$ <p><span style="border: 1px solid red; padding: 2px;">True</span> Lie</p>

<p><b>Problem 3</b></p> $\lim_{x \rightarrow 0} \frac{1}{x+4} - \frac{1}{4x}$	$\lim_{x \rightarrow 0} \frac{\frac{4}{4} \cdot \frac{1}{x+4} - \frac{1}{4} \cdot \frac{x+4}{x+4}}{x}$ $= \lim_{x \rightarrow 0} \frac{\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)}}{x}$ $= \lim_{x \rightarrow 0} \frac{\frac{-x+4}{4(x+4)}}{x}$ $= \lim_{x \rightarrow 0} \frac{-x+4}{4(x+4)} \cdot \frac{1}{x}$ $= \lim_{x \rightarrow 0} \frac{-x+4}{4(x+4)} \cdot \frac{1}{x}$ $= \text{dne}$ <p>True <span style="border: 1px solid red; padding: 2px;">Lie</span></p> <p><b>The subtraction sign was not distributed through the numerator.</b></p>	$\lim_{x \rightarrow 0} \frac{\frac{4}{4} \cdot \frac{1}{x+4} - \frac{1}{4} \cdot \frac{x+4}{x+4}}{x}$ $= \lim_{x \rightarrow 0} \frac{\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)}}{x}$ $= \lim_{x \rightarrow 0} \frac{-x}{4(x+4)}$ $= \lim_{x \rightarrow 0} \frac{-x}{4(x+4)} \cdot \frac{1}{x}$ $= \lim_{x \rightarrow 0} \frac{1}{4(x+4)} \cdot \frac{1}{1}$ $= \frac{1}{8}$ <p>True <span style="border: 1px solid red; padding: 2px;">Lie</span></p> <p><b>When evaluating the limit, 4 times 4 was incorrectly calculated as 8.</b></p>	$\lim_{x \rightarrow 0} \frac{\frac{4}{4} \cdot \frac{1}{x+4} - \frac{1}{4} \cdot \frac{x+4}{x+4}}{x}$ $= \lim_{x \rightarrow 0} \frac{\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)}}{x}$ $= \lim_{x \rightarrow 0} \frac{-x}{4(x+4)}$ $= \lim_{x \rightarrow 0} \frac{-x}{4(x+4)} \cdot \frac{1}{x}$ $= \lim_{x \rightarrow 0} \frac{-1}{4(x+4)} \cdot \frac{1}{1}$ $= -\frac{1}{16}$ <p><span style="border: 1px solid red; padding: 2px;">True</span> Lie</p>
<p><b>Problem 4</b></p> $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$	$\lim_{h \rightarrow 0} \frac{9 + 6h + h^2 - 9}{h}$ $= \lim_{h \rightarrow 0} \frac{6h + h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{h(6+h)}{h}$ $= \lim_{h \rightarrow 0} (6+h)$ $= 6$ <p><span style="border: 1px solid red; padding: 2px;">True</span> Lie</p>	$\lim_{h \rightarrow 0} \frac{9 + h^2 - 9}{h}$ $= \lim_{h \rightarrow 0} \frac{h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{1}{h}$ $= \text{dne}$ <p>True <span style="border: 1px solid red; padding: 2px;">Lie</span></p> <p><b>The expansion of <math>(3+h)^2</math> is incorrect as it is missing the <math>6h</math> term.</b></p>	$\lim_{h \rightarrow 0} \frac{9 + 6h + h^2 - 9}{h}$ $= \lim_{h \rightarrow 0} \frac{6h + h^2}{h}$ $= \lim_{h \rightarrow 0} \frac{6h+h}{h}$ $= 0$ <p>True <span style="border: 1px solid red; padding: 2px;">Lie</span></p> <p><b>The simplification of the second step is incorrect as both <math>h</math> terms in the numerator were not considered.</b></p>

