CALC

1998 AP Calculus AB
Question 6

Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

(a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.

$$\frac{d}{dx}2y^3 + \frac{d}{dx}\underbrace{6x^2y}_{P.R.} - \frac{d}{dx}12x^2 + \frac{d}{dx}6y = \frac{d}{dx}1$$

$$6y^2\frac{dy}{dx} + \underbrace{12x\cdot y + 6x^2\cdot\frac{dy}{dx}}_{\text{Product Rule}} - 24x + 6\frac{dy}{dx} = 0 \quad +1$$

$$6y^2\frac{dy}{dx} + 6x^2\cdot\frac{dy}{dx} + 6\frac{dy}{dx} = -12xy + 24x$$

$$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = -12xy + 24x$$

$$\frac{dy}{dx} = \frac{-12xy + 24x}{6y^2 + 6x^2 + 6} = \frac{6(-2xy + 4x)}{6(y^2 + x^2 + 1)} = \frac{-2xy + 4x}{y^2 + x^2 + 1} \quad +1$$

(b) Write an equation of each horizontal tangent line to the curve.

$$\frac{dy}{dx} = 0$$

$$-2xy + 4x = 0 \quad +1$$

$$-2x(y - 2) = 0$$

$$-2x = 0 \qquad y - 2 = 0$$

$$\boxed{x = 0 \qquad y = 2} \rightarrow +1$$

$$2y^3 + 6x^2y - 12x^2 + 6y = 1$$
$$2y^3 + 6(0)^2y - 12(0)^2 + 6y = 1$$
$$2y^3 + 6y - 1 = 0$$

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$$y \approx 0.165$$

$$2y^3 + 6x^2y - 12x^2 + 6y = 1$$
$$2(2)^3 + 6x^2(2) - 12x^2 + 6(2) = 1$$
$$16 + 12x^2 - 12x^2 + 12 = 1$$
$$28 \neq 1$$

No solution

$$\therefore y \neq 2 \quad +1$$

POT $(0, 0.165)$

SOT $= 0$

Tangent line: $y - 0.165 = 0(x - 0)$ $+1$

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(c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x– and y–coordinates of point P.

POT $(0,0)$

SOT $= -1$

Tangent line: $y - 0 = -1(x - 0)$

$y = -x$

+1

$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$ +1

$-2x^3 - 6x^3 - 12x^2 - 6x = 1$

$-8x^3 - 12x^2 - 6x - 1 = 0$

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$x = -1/2$

$2y^3 + 6\left(-\frac{1}{2}\right)^2 y - 12\left(-\frac{1}{2}\right)^2 + 6y - 1 = 0$

$2y^3 + \frac{6}{4}y - \frac{12}{4} + 6y - 1 = 0$

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$y = 1/2$

P has coordinates $\left(-\frac{1}{2}, \frac{1}{2}\right)$

+1