Added = 8 g/min

Lost = $(t+1)^{1/2}$ g/min

2000 AP® CALCULUS AB

Problem #4

$A(0) = 30$ g

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) $\int_0^3 \sqrt{t+1}\,dt$ +1 +1

$= 4.667$ gallons +1

(b) $\int_0^t \left[8 - \sqrt{t+1}\right] dt = T(t) - T(0)$

$\int_0^3 \left[8 - \sqrt{t+1}\right] dt = T(3) - T(0)$

$19.333 = T(3) - 30$

$T(3) = 49.333$ gallons +1

(c) $\int_0^t \left[8 - \sqrt{t+1}\right] dt = A(t) - A(0)$

$8t - \int_0^t \sqrt{t+1}\,dt = A(t) - 30$

$A(t) = 30 + 8t - \int_0^t \sqrt{t+1}\,dx$

+1 +1

(d) CV: $t = 63$

$A'(t) = A'(0) + \left[8 - \sqrt{t+1}\right]$ OPTIONAL

$A'(t) = 30' + 8 - \sqrt{t+1}$

$A'(t) = 8 - \sqrt{t+1} = 0$ +1

$\sqrt{t+1} = 8$ OPTIONAL

$t+1 = 64$

$t = 63$ +1

t	$A(t)$
0	30
63	193.333 ✓
120	103.333

+1

The amount of water in the tank is at a maximum at $t = 63$ minutes. The maximum water is 193.333 gallons.