

2000 AP® CALCULUS AB

Question 5

*Product Product*Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

$$\frac{d}{dx}(xy^2) - \frac{d}{dx}(x^3y) = \frac{d}{dx} 6$$

$$(1)y^2 + x^2y \frac{dy}{dx} - [3x^2y + x^3(1)\frac{dy}{dx}] = 0 \quad (+1)$$

$$y^2 - 3x^2y + 2xy \frac{dy}{dx} - x^3 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

- (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

POT $(1, -2), (1, 3)$

SOT: $\frac{dy}{dx}|_{(1,-2)} = 2$

$\frac{dy}{dx}|_{(1,3)} = 0$

$(1)y^2 - (1)^3y = 6$

$+1 \quad y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y-3=0 \quad \left\{ \begin{array}{l} y+2=0 \\ y=3 \end{array} \right.$

$y=-2$

$\frac{dy}{dx}|_{(1,-2)} = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3}$

$= \frac{3(-2) - 4}{-4 - 1}$

$= \frac{-6 - 4}{-5}$

$= \frac{-10}{-5}$

$= 2$

$\frac{dy}{dx}|_{(1,3)} = \frac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3}$

$= \frac{3 \cdot 3 - 9}{6 - 1}$

$= \frac{0}{5}$

$\frac{dy}{dx}|_{(1,3)} = 0$

Tangent Line

$y + 2 = 2(x - 1)$

(+1)

Tangent Line

$y - 3 = 0(x - 1)$

(+1)

- (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

$\frac{dy}{dx} = \text{und}$

$0 = 2xy - x^3$

$0 = x(2y - x^2)$

$0 = x \quad \left\{ \begin{array}{l} 2y - x^2 = 0 \\ 2y = x^2 \\ y = \frac{x^2}{2} \end{array} \right.$

Does $x=0$ work?

$0 \cdot y^2 - 0^3 \cdot y = 6$

$0 = 6$

Nope

$xy^2 - x^3y = 6$

$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6 \quad (+1)$

$\frac{x \cdot x^4}{4} - \frac{x^5}{2} = 6$

$\frac{x^5}{4} - \frac{2x^5}{4} = 6$

$-\frac{x^5}{4} = 6$

$-x^5 = 24$

$x^5 = -24$

$x = -\sqrt[5]{24}$

(+1)