2000 AP® CALCULUS AB

Question 5

Product Product

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

$$\frac{d}{dx}(xy^2) - \frac{d}{dx}(x^3y) = \frac{d}{dx}6$$

$$(1)\cdot y^2 + x\cdot 2y\frac{dy}{dx} - \left[3x^2y + x^3(1)\frac{dy}{dx}\right] = 0 \quad +1$$

$$y^2 - 3x^2y + 2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3} \quad +1$$

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

PoT $(1,-2), (1,3)$ $\qquad$ SoT : $\left.\frac{dy}{dx}\right|_{(1,-2)} = 2$ $\qquad$ $\left.\frac{dy}{dx}\right|_{(1,3)} = 0$

$(1)y^2 - (1)^3y = 6$ $+1$ $y^2 - y - 6 = 0$ $(y-3)(y+2) = 0$ $y - 3 = 0$, $y = 3$; $y + 2 = 0$, $y = -2$ $+1$	$\left.\frac{dy}{dx}\right\|_{(1,-2)} = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3}$ $= \frac{3(-2) - (4)}{-4 - 1}$ $= \frac{-6-4}{-5}$ $= \frac{-10}{-5}$ $= 2$	$\left.\frac{dy}{dx}\right\|_{(1,3)} = \frac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3}$ $= \frac{3\cdot 3 - 9}{6 - 1}$ $= \frac{0}{5}$ $\left.\frac{dy}{dx}\right\|_{(1,3)} = 0$

Tangent Line $\qquad$ Tangent Line

$y + 2 = 2(x - 1)$ $+1$ $\qquad$ $y - 3 = 0(x - 3)$ $+1$

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

$\frac{dy}{dx} = $ und

$+1$ $0 = 2xy - x^3$

$0 = x(2y - x^2)$

$0 = x$; $2y - x^2 = 0$, $2y = x^2$, $y = \frac{x^2}{2}$

Does $x=0$ work?

$xy^2 - x^3y = 6.$

$0\cdot y^2 \cdot 0^3 \cdot y = 6$

$0 = 6$

nope

$$xy^2 - x^3y = 6$$

$$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6 \quad +1$$

$$\frac{x\cdot x^4}{4} - \frac{x^5}{2} = 6$$

$$\frac{x^5}{4} - \frac{2x^5}{4} = 6$$

$$\frac{-x^5}{4} = 6$$

$$-x^5 = 24$$

$$x^5 = -24$$

$$x = -\sqrt[5]{24} \quad +1$$