

2000 AP Calculus AB  
Question 6

Consider the differential equation  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ .

- (a) Find a solution  $y = f(x)$  to the differential equation satisfying  $f(0) = \frac{1}{2}$ .

$$\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$$

$$\int e^{2y} dy = \int 3x^2 dx$$

$$\frac{1}{2} e^{2y} = x^3 + C$$

at  $(0, \frac{1}{2})$

$$\frac{1}{2} e^{2(\frac{1}{2})} = (0)^3 + C$$

$$\frac{1}{2} e = C$$

$$\frac{1}{2} e^{2y} = x^3 + \frac{1}{2} e$$

$$e^{2y} = 2x^3 + e$$

$$2y = \ln(2x^3 + e)$$

$$y = \frac{1}{2} \ln(2x^3 + e)$$

$$f(x) = \frac{1}{2} \ln(2x^3 + e)$$

- (b) Find the domain and range of the function  $f$  found in part (a).

Domain  $(\sqrt[3]{\frac{-e}{2}}, \infty)$

Range  $(-\infty, \infty)$

Argument  $> 0$

$$2x^3 + e > 0$$

$$2x^3 > -e$$

$$x^3 > -\frac{e}{2}$$

$$x > \sqrt[3]{\frac{-e}{2}}$$

Not part of answer

$$y = \log_a x$$
