2001 AP® CALCULUS AB
Question 4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

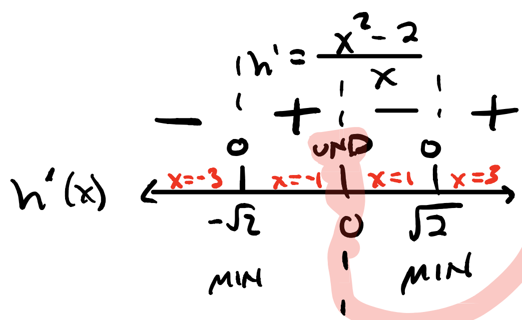
(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(a) $h'(x) = \dfrac{x^2-2}{x}$

$0 = \dfrac{x^2-2}{x}$

Horizontal tangent	Undefined
$0 = x^2 - 2$	$x = 0$
$2 = x^2$	
$\pm\sqrt{2} = x$	

$\therefore$ h has horizontal tangents at $x = -\sqrt{2}$ and $x = \sqrt{2}$



$\therefore$ h has Local mins at $x = -\sqrt{2}$ and $x = \sqrt{2}$

(b) $h'(x) = \dfrac{x^2}{x} - \dfrac{2}{x} = x - 2x^{-1}$

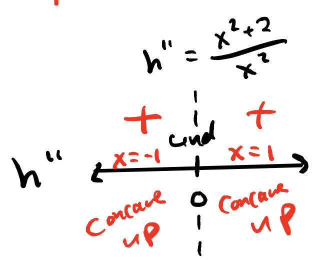
$h''(x) = 1 + 2x^{-2}$

$h''(x) = \dfrac{x^2}{x^2} + \dfrac{2}{x^2}$

$h''(x) = \dfrac{x^2+2}{x^2}$

$0 = \dfrac{x^2+2}{x^2}$

horizontal tangent	undefined
$0 = x^2 + 2$	$x = 0$
$-2 = x^2$	
$\pm\sqrt{-2} = x$	
no solution	



$\therefore$ h is Concave up on $(-\infty, 0) \cup (0, \infty)$

(c)

PoT	SoT
$(4, -3)$	$h'(4) = \dfrac{(4)^2-2}{4}$
	$= \dfrac{16-2}{4}$
	$= \dfrac{14}{4}$
	$h'(4) = 7/2$

Tangent line

$y + 3 = \frac{7}{2}(x - 4)$

(D) The tangent line to the graph h lies below the graph of h for $x > 4$ because h is concave up on $(0, \infty)$