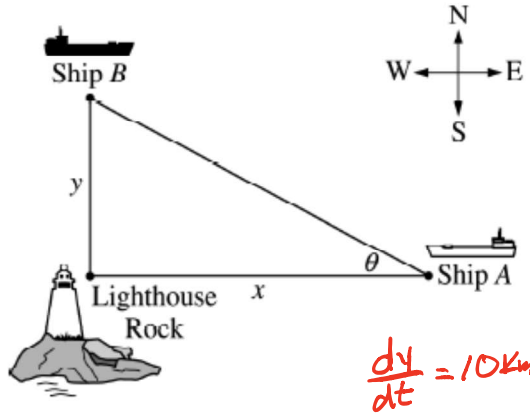
2002 AP® CALCULUS AB (Form B)

Question #6

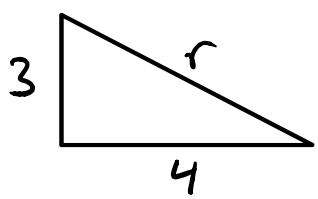


$\frac{dx}{dt} = -15 \text{ Km/hr}$

$\frac{dy}{dt} = 10 \text{ Km/hr}$

Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.



PT (3-4-5)

Distance = 5 Km +1

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

FIND $\left.\frac{dr}{dt}\right|_{(4,3)}$ $r=5$

$\frac{dx}{dt} = -15 \text{ Km/hr}$

$\frac{dy}{dt} = 10 \text{ Km/hr}$

$x^2 + y^2 = r^2$ +1

$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt} r^2$

$2x \cdot \frac{dx}{dt} + 2y\frac{dy}{dt} = 2r \cdot \frac{dr}{dt}$ +2

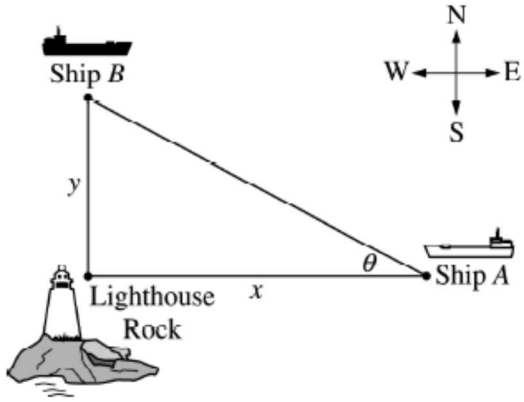
$2(4)\cdot(-15) + 2(3)(10) = 2(5)\cdot\frac{dr}{dt}$

$-120 + 60 = 10 \cdot \frac{dr}{dt}$

$-60 = 10 \cdot \frac{dr}{dt}$

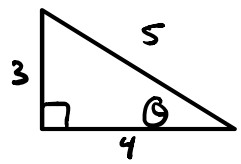
$-6 = \frac{dr}{dt}$

$\frac{dr}{dt} = -6 \text{ Km/hr}$ +1



Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.



$\cos\theta = \frac{4}{5}$

$\sec\theta = \frac{5}{4}$

$\frac{dx}{dt} = -15$ km/hr

$\frac{dy}{dt} = 10$ km/hr

$$\tan\theta = \frac{y}{x} \quad +1$$

$$\frac{d}{dt}\tan\theta = \frac{d}{dt}\left(\frac{y}{x}\right)$$

$$\sec^2(\theta)\cdot\frac{d\theta}{dt} = \frac{\frac{dy}{dt}\cdot x - y\cdot\frac{dx}{dt}}{x^2}$$ Quotient Rule +2

$$\left(\frac{5}{4}\right)^2\cdot\frac{d\theta}{dt} = \frac{10\cdot(4)-(3)(-15)}{(4)^2}$$

$$\frac{25}{16}\cdot\frac{d\theta}{dt} = \frac{40+45}{16}$$

$$\frac{d\theta}{dt} = \frac{\cancel{85}^{17}}{\cancel{16}}\cdot\frac{\cancel{16}}{\cancel{25}_{5}}$$

$$\frac{d\theta}{dt} = \frac{17}{5} \text{ RADIANS/hr} \quad +1$$