2002 AP® CALCULUS AB (Form B)

Problem #2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

$P(0) = 50$

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

(a) $P'(9) = -0.646$ +1

The amount of pollutant is <u>not</u> increasing at $t=9$ because $P'(t) < 0$ at $t=6$.

(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$

Not required

$1 = 3e^{-0.2\sqrt{t}}$

$\frac{1}{3} = e^{-0.2\sqrt{t}}$

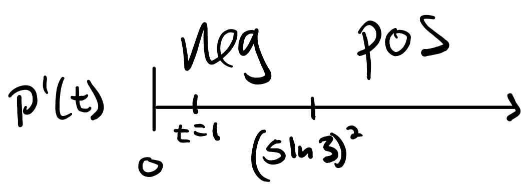
$\ln\frac{1}{3} = -0.2\sqrt{t}$

$\ln 1 - \ln 3 = -\frac{1}{5}\sqrt{t}$

$0 - \ln 3 = -\frac{1}{5}\sqrt{t}$

$5\ln 3 = \sqrt{t}$

+1 $t = (5\ln 3)^2 \approx 30.174$



$P'(t) < 0$ on $0 < t < 30.174$ +1

and $P'(t) > 0$ on $t > 30.174$

$\therefore$ There is a minimum at $t = 30.174$ days

(c) $\int_0^t P'(t)\,dt = P(t) - P(0)$

$\int_0^{(5\ln 3)^2} P'(t)\,dt = P(5\ln 3)^2 - 50$

$-14.896 = P(5\ln 3)^2 - 50$

$P(5\ln 3)^2 = 35.104$

The lake is safe at it minimum because $P(5\ln 3)^2 = 35.104 < 40$.

(D) POT(0,50) SOT = −2 Tangent

$P'(0) = 1 - 3e^{-0.2\sqrt{0}}$

$= 1 - 3(1)$

$P'(0) = -2$ +1

$y - 50 = -2(x - 0)$

$y = -2x + 50$

Safe

$40 = -2x + 50$

$-10 = -2x$

$5 = x$

The model predicts the lake will be safe when $t = 5$ hours

+1