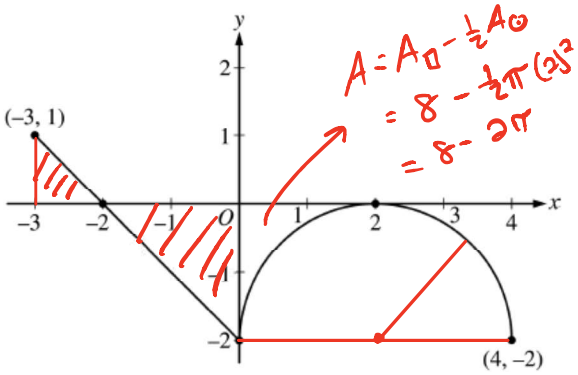
2003 AP® CALCULUS AB

Problem #4



Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

a. On what intervals, if any, is f increasing. Justify your reasoning.

f is increasing on $[-3, 2)$ b/c $f' > 0$ on $[-3, 2)$

b. Find the x – coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

f has inflection points at $x = 0$ and $x = 2$ because
f' changes from decreasing to increasing at $x = 0$ and
f' changes from increasing to decreasing at $x = 2$.

c. Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

$y - 3 = -2(x - 0)$

d. Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

$f(0) = 3$

$\int_{-3}^{0} f'(x)\,dx = f(0) - f(-3)$

$\frac{1}{2} + (-2) = 3 - f(-3)$

$-1.5 = 3 - f(-3)$

$-4.5 = -f(-3)$

$4.5 = f(-3)$

$f(0) = 3$

$\int_{0}^{4} f'(x)\,dx = f(4) - f(0)$

$-(8 - 2\pi) = f(4) - 3$

$-8 + 2\pi = f(4) - 3$

$-5 + 2\pi = f(4)$