Name **Date** **Class**

Homework 4.3 part 2
2005 AP® CALCULUS AB
(Form B)
Question 5

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y - x}$.

$\frac{d}{dx}(y^2) = \frac{d}{dx}(2 + xy)$

$2y\frac{dy}{dx} = 1 \cdot y + x \cdot \frac{dy}{dx}$ +1

Product Rule

$2y\frac{dy}{dx} - x \cdot \frac{dy}{dx} = y$

$\frac{dy}{dx}(2y - x) = y$ +1

$\frac{dy}{dx} = \frac{y}{2y - x}$

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

$\frac{dy}{dx} = \frac{1}{2}$ $\therefore$ $\frac{y}{2y - x} = \frac{1}{2}$ +1

$2y = 2y - x$

$0 = -x$

$0 = x$

$y^2 = 2 + xy$

$y^2 = 2 + (0)y$

$y^2 = 2$

$y = \pm\sqrt{2}$

+1

$(0, -\sqrt{2})$, $(0, \sqrt{2})$

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Consider the curve given by $y^2 = 2 + xy$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

$$\frac{dy}{dx} = \frac{y}{2y - x}$$

$\frac{dy}{dx} = 0$ if curve is horizontal

$\therefore y = 0$ +1

$y^2 = 2 + xy$

$0^2 = 2 + x(0)$

$0 \neq 2$ False

} +1

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

$$\frac{d}{dt}(y^2) = \frac{d}{dt}(2 + xy)$$

$$2y \cdot \frac{dy}{dt} = \frac{dx}{dt} y + x \cdot \frac{dy}{dt}$$ +1

Product Rule

$y^2 = 2 + xy$

$(3)^2 = 2 + x \cdot 3$

$9 = 2 + 3x$

$7 = 3x$

$\frac{7}{3} = x$ +1

$2(3)(6) = \frac{dx}{dt}\Big|_{t=5}(3) + \left(\frac{7}{3}\right)(6)$

$36 = 3 \cdot \frac{dx}{dt}\Big|_{t=5} + 14$

$22 = 3 \cdot \frac{dx}{dt}\Big|_{t=5}$

$\frac{22}{3} = \frac{dx}{dt}\Big|_{t=5}$ +1