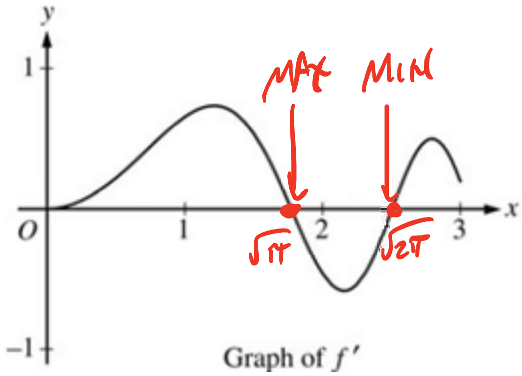
2006 AP® CALCULUS AB
(Form B)
Question 2
(A Modified Version)

Let f be the function defined for $x \geq 0$ with $f(2) = 5.623$ and $f\,'$, the first derivative of f, given by $f\,'(x) = e^{(-x/4)}\sin\left(x^2\right)$. The graph of $f\,'$ is shown to the right.



Graph of $f\,'$

(a) Use the graph of $f\,'$ to determine whether the graph of f is concave up, down or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

(b) On the interval $0 < x < 3$, at what x – value(s) does the graph of f have a relative maximum? A relative minimum? Justify your answers.

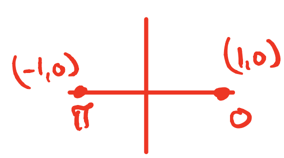
(c) Write an equation for the tangent line to the graph of f at $x = 2$. Will the tangent line be above or below the graph of f for $1.5 < x < 2$? Give a reason for your answer.

(a) f is concave down on $(1.7, 1.9)$ because f' is decreasing on this interval.

(b) $f'(x) = e^{-x/4}\sin(x^2)$

horizontal tangent

$0 = e^{-x/4}$ no solution	$0 = \sin(x^2)$ $x^2 = 0$, $x = 0$; $x^2 = \pi$, $x = \sqrt{\pi}$; $x^2 = 2\pi$, $x = \sqrt{2\pi}$



f has a relative max at $x = \sqrt{\pi}$ because f' changes from + to − $x = \sqrt{\pi}$.
f has a relative min at $x = \sqrt{2\pi}$ because f' changes from − to + $x = \sqrt{2\pi}$.

(c)

PoT	SoT	tangent line
$(2, 5.623)$	$f'(2) = e^{-2/4}\sin(2)^2$ $f'(2) = e^{-\frac{1}{2}}\sin(4)$	$y - 5.623 = e^{-1/2}\sin(4)(x-2)$

The tangent line at $x = 2$ will be above the graph of f b/c f is concave down on $(1.5, 2)$