Unit 3.6 Calculating Higher-Order Derivatives

2006 AP® CALCULUS AB

Question 6

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions: $f(0)=2$, $f'(0)=-4$, and $f''(0)=3$.

(a) The function g is given by $g(x)=e^{ax}+f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

$g'(x) = ae^{ax} + f'(x)$

$g'(0) = ae^{a(0)} + f'(0)$

$= ae^{0} + (-4)$

$g'(0) = a - 4$

$g''(x) = a^2e^{ax} + f''(x)$

$g''(0) = a^2e^{a\cdot 0} + f''(0)$

$= a^2\cdot e^0 + 3$

$g''(0) = a^2 + 3$

(b) The function h is given by $h(x)=\cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x=0$.

PoT	SoT
$h(0) = \cos(k\cdot 0)\, f(0)$	$h'(x) = -k\sin(kx)\cdot f(x) + \cos(kx)\cdot f'(x)$
$= \cos(0)\cdot 2$	$h'(0) = -k\sin(k\cdot 0)\cdot f(0) + \cos(k\cdot 0)\, f'(0)$
$= 1\cdot 2$	$h'(0) = -k\sin(0)\cdot f(0) + \cos(0)\cdot f'(0)$
$h(0) = 2$	$= 0\cdot f(0) + 1\cdot f'(0)$
	$= 0 + -4$
	$h'(0) = -4$

tangent

$y - 2 = -4(x - 0)$