2007 AP® CALCULUS AB
Question 6

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

(c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

$\therefore y=0$

(a) $f'(x) = \frac{1}{2}k \cdot x^{-1/2} - \frac{1}{x} = \frac{1}{2}kx^{-1/2} - x^{-1}$

$f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$

(b) $0 = \frac{k}{2\sqrt{x}} - \frac{1}{x}$

$\boxed{@x=1}$ $0 = \frac{k}{2\sqrt{1}} - \frac{1}{1}$

$0 = \frac{k}{2} - 1$

$1 = \frac{k}{2}$

$2 = k$

f has a critical point at $x=1$ when $k = 2$.

2nd Derivative Test

$f''(1) = \frac{-\frac{1}{2}}{\sqrt{1^3}} + \frac{1}{(1)^2} = -\frac{1}{2} + 1 > 0$

concave up, min

$\therefore f$ has a relative min at $x=1$ by the 2nd Derivative Test.

(c)

1. $f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$

$0 = -\frac{1}{4}kx^{-3/2} + x^{-2}$

$\frac{1}{4}kx^{-3/2} = x^{-4/2}$

$k = 4x^{-1/2}$

2. $f(x) = k\sqrt{x} - \ln x$

$0 = k\sqrt{x} - \ln x$

$\frac{\ln x}{\sqrt{x}} = k$

3. $\sqrt{x} \cdot \frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}} \cdot \sqrt{x}$

$4 = \ln x$

$e^4 = x$

4. $k = \frac{\ln(e^4)}{\sqrt{e^4}}$

$k = \frac{4}{e^2}$