Notes 4.2 – Implicit Differentiation in AP Calculus Free Responses

2008 AP® CALCULUS AB
(Form B)
Question 6

Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(a) Show that $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$.

$$\frac{d}{dx}x^2 + \frac{d}{dx}2x + \frac{d}{dx}y^4 + \frac{d}{dx}4y = \frac{d}{dx}5$$

$$2x + 2 + 4y^3\frac{dy}{dx} + 4\frac{dy}{dx} = 0$$ +1

$$4y^3\frac{dy}{dx} + 4\frac{dy}{dx} = -2x-2$$

$$\frac{dy}{dx}(4y^3+4) = -2x-2$$

$$\frac{dy}{dx} = \frac{-2x-2}{4y^3+4}$$

$$= \frac{-2(x+1)}{4(y^3+1)}$$

$$\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$$

+1

(b) Write an equation for the line tangent to the curve at the point $(-2, 1)$.

POT $(-2,1)$	SOT: $\frac{dy}{dx}\Big\vert_{(-2,1)} = \frac{1}{4}$	Tangent Line
	$\frac{dy}{dx}\Big\vert_{(-2,1)} = \frac{-(-2+1)}{2(1^3+1)}$ $= \frac{-(-1)}{2(2)}$ $\frac{dy}{dx}\Big\vert_{(-2,1)} = \frac{1}{4}$ +1	$y-1 = \frac{1}{4}(x+2)$ +1

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Consider the closed curve in the xy-plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

(c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.

vertical tangent $\Rightarrow \frac{dy}{dx} = \text{und}$

$0 = 2(y^3+1)$

$0 = y^3 + 1$

$-1 = y^3$

$-1 = y$ +1

$x^2 + 2x + y^4 + 4y = 5$

$x^2 + 2x + (-1)^4 + 4(-1) = 5$ +1

$x^2 + 2x + 1 - 4 = 5$

$x^2 + 2x - 3 = 5$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x + 4 = 0$, $x = -4$; $x - 2 = 0$, $x = 2$

The tangent line is vertical at $(-4,-1)$ and $(2,-1)$ +1

(d) Is it possible for this curve to have a horizontal tangent at points where it intersects the x-axis? Explain your reasoning.

d) Horizontal tangent $\Rightarrow \frac{dy}{dx} = 0$

$-(x+1) = 0$

$x + 1 = 0$

$x = -1$

Horizontal tangent at $x = -1$ +1

crosses x-axis when $y = 0$

$x^2 + 2x + 0^4 + 4(0) = 5$

$x^2 + 2x = 5$

Does $x = -1$ make this true?

$(-1)^2 + 2(-1) \neq 5$

$1 - 2 \neq 5$

$-1 \neq 5$

no the curve cannot have a horizontal tangent where it crosses the x-axis. +1