

## 2015 AP® CALCULUS AB

## Question 6

Consider the curve given by the equation  $y^3 - xy = 2$ . It can be shown that  $\frac{dy}{dx} = \frac{y}{3y^2 - x}$ .

- (a) Write an equation for the line tangent to the curve at the point  $(-1, 1)$ .

POT $(-1, 1)$	SOT $\frac{dy}{dx} _{(-1,1)} = \frac{1}{4}$	Tangent line
	$\frac{dy}{dx} _{(-1,1)} = \frac{1}{3(1)^2 - (-1)} = \frac{1}{3+1} = \frac{1}{4}$	$y - 1 = \frac{1}{4}(x + 1)$ <span style="color: blue;">+1</span>
	$\frac{dy}{dx} _{(-1,1)} = \frac{1}{4}$ <span style="color: blue;">+1</span>	

- (b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

$$\begin{aligned} \text{Step 1} \quad \frac{dy}{dx} &= \text{und} & \text{Step 2} \quad y^3 - xy = 2 \quad (\text{original EQ}) \\ 3y^2 - x &= 0 \quad +1 & y^3 - (3y^2)y = 2 \quad +1 \\ 3y^2 &= x & y^3 - 3y^3 = 2 \\ \text{Step 3} \quad 3(-1)^2 &= x & -2y^3 = 2 \\ 3 &= x & y^3 = -1 \\ && y = -1 \end{aligned}$$

The tangent line to the curve is vertical at  $(3, -1)$

+1

## 2015 AP® CALCULUS AB

## Question 6

Consider the curve given by the equation  $y^3 - xy = 2$ . It can be shown that  $\frac{dy}{dx} = \frac{y}{3y^2 - x}$ .

- (c) Evaluate  $\frac{d^2y}{dx^2}$  at the point on the curve where  $x = -1$  and  $y = 1$ .

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{\frac{dy}{dx}(3y^2-x) - y \cdot (6y \cdot \frac{dy}{dx} - 1)}{(3y^2-x)^2} \quad +2 \\ \left. \frac{d^2y}{dx^2} \right|_{(-1,1)} &= \frac{\frac{1}{4}(3(1)^2 - (-1)) - (1)(6(1)\frac{1}{4} - 1)}{(3(1)^2 - (-1))^2} \\ &= \frac{\frac{1}{4}(4) - \left(\frac{3}{2} - \frac{1}{2}\right)}{(4)^2} \\ &= \frac{1 - \frac{1}{2}}{16} \\ &= \frac{\frac{1}{2}}{16} \quad \cdot 2 \\ &= \frac{1}{32} \quad +1\end{aligned}$$

$$\left. \frac{dy}{dx} \right|_{(-1,1)} = \frac{1}{4} \text{ (Part a)}$$