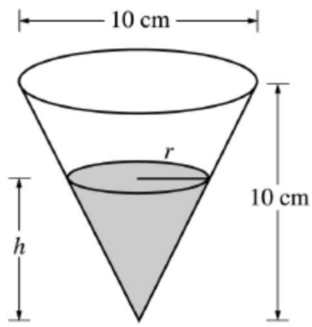
Question #5



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth, h, is changing at the rate of $-\frac{3}{10}$ centimeters per hour.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Suppose you were asked to find the rate of change of the volume of water in the container when the depth of the water is 5 cm. Explain why the formula $V = \frac{1}{3}\pi r^2 h$ could not be implicitly differentiated with respect to time in order to determine this rate? What would have to be done in order to find this rate?

$$\frac{d}{dt}V = \frac{d}{dt}\left(\frac{1}{3}\pi r^2 h\right)$$

Product Rule

$$\frac{dV}{dt} = \frac{2}{3}\pi r\frac{dr}{dt}\cdot h + \frac{1}{3}\pi r^2\cdot\frac{dh}{dt}$$

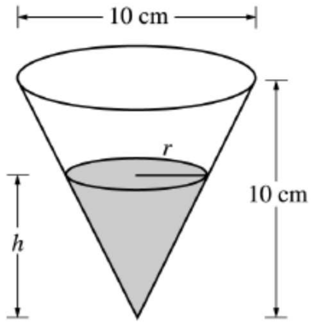
$$\left.\frac{dV}{dt}\right|_{\substack{h=5\\ r=2.5}} = \frac{2}{3}\pi(2.5)\frac{dr}{dt}(5) + \frac{1}{3}\pi(2.5)^2\cdot\left(\frac{-3}{10}\right)$$

When we differentiate V with respect to time, we end up with the variable $\frac{dr}{dt}$, but we don't know what $\frac{dr}{dt}$ is equal to. ∴ we have two variables $\frac{dV}{dt}$ and $\frac{dr}{dt}$ and can't find the value of $\frac{dV}{dt}$. IN order to find $\frac{dV}{dt}$, we need to remove r by substituting with $\frac{1}{2}h$.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate correct units of measure.

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$$
$$V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h$$
$$V = \frac{1}{12}\pi h^3$$

$$\frac{d}{dt}V = \frac{d}{dt}\frac{1}{12}\pi h^3$$
$$\frac{dV}{dt} = \frac{1}{4}\pi h^2\cdot\frac{dh}{dt}$$
$$\left.\frac{dV}{dt}\right|_{h=5} = \frac{1}{4}\pi(5)^2\cdot\left(\frac{-3}{10}\right)$$
$$\left.\frac{dV}{dt}\right|_{h=5} = -\frac{1}{4}\pi\,\cancel{25}^{\,5}\cdot\frac{3}{\cancel{10}_{\,2}}$$
$$\left.\frac{dV}{dt}\right|_{h=5} = \frac{-15\pi}{8}\ \text{cm}^3/\text{hr}$$



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(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(c) The surface area of the exposed water is changing at a rate of 9π cm^2 per hour when the depth of the water is 4 cm. At what rate is the radius of the exposed water changing at this point in time? Indicate correct units of measure.

$$\left.\frac{dS}{dt}\right|_{\substack{h=4\\ r=2}} = \frac{-9\pi \text{ cm}^2}{1 \text{ hr}}$$

FIND $\frac{dr}{dt}$

$$S = \pi r^2$$

$$\frac{d}{dt} S = \frac{d}{dt} \pi r^2$$

$$\frac{dS}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

$$-9\pi = 2\pi(2)\frac{dr}{dt}$$

$$-\frac{9}{4} \text{ cm/hr} = \frac{dr}{dt}$$