Skill Builder: Topic 7.8 – Exponential Models with Differential Equations

APClassroom Assignment Hw 7.8

1. If $\frac{dy}{dx} = 4y$ and if y=4 when x=0, then y=

(A) $4e^{4x}$
(B) e^{4x}
(C) $3+e^{4x}$
(D) $4+e^{4x}$
(E) $2x^2 + 4$

C

$y = Ce^{kx}$

$y = 4e^{4x}$

$\frac{dy}{dx} = k \cdot y$

2. Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

(A) 3ln3/ln2
(B) 2ln3/ln2
(C) ln3/ln2
(D) ln(27/2)
(E) ln(9/2)

$k = 3$

$y = Ce^{kx}$

$y = Ce^{kx}$

Doubles every 3 hrs → $2 = 1 \cdot e^{k(3)}$

$2 = e^{3k}$

$\ln 2 = 3k$

$\frac{1}{3}\ln 2 = k$

$\ln 2^{1/3} = k$

$3 = 1 \cdot e^{\ln 2^{1/3} \cdot t}$

$3 = e^{\ln 2^{t/3}}$

$3 = 2^{t/3}$

$\ln 3 = \ln 2^{t/3}$

$\ln 3 = \frac{t}{3} \cdot \ln 2$

$\frac{\ln 3}{\ln 2} = \frac{t}{3}$

$\frac{3\ln 3}{\ln 2} = t$

3. Population y grows according to the equation $\frac{dy}{dt} = ky$, where k is a constant and t is measured in years. If the population doubles every 10 years, then the value of k is

(A) 0.069
(B) 0.200
(C) 0.301
(D) 3.322
(E) 5.000

$y = Ce^{kt}$

$2 = 1 \cdot e^{k(10)}$

$2 = e^{10k}$

$\ln 2 = 10k$

$\frac{1}{10}\ln 2 = k$

(0, 2) (2, 3.5)

4. A puppy weighs 2.0 pounds at birth and 3.5 pounds two months later. If the weight of the puppy during its first 6 months is increasing at a rate proportional to its weight, then how much will the puppy weigh when it is 3 months old?

(A) 4.2 pounds
(B) 4.6 pounds
(C) 4.8 pounds
(D) 5.6 pounds
(E) 6.5 pounds

① $\frac{dy}{dx} = ky$

$y = Ce^{kx}$

$(0,2) \rightarrow y = 2e^{kx}$

② Find k use $(2, 3.5)$

$3.5 = 2e^{k(2)}$

$\frac{3.5}{2} = e^{2k}$

$\ln(1.75) = 2k$

$\frac{1}{2}\ln(1.75) = k$

$y = 2e^{(\ln 1.75)^{\frac{1}{2}} \cdot x}$

$y = 2(1.75^{\frac{x}{2}})$

③ $y = 2(1.75)^{\frac{x}{2}}$

$y(3) \approx 4.630$ $^{3/2}$

5. During a certain epidemic, the number of people that are infected at any time increases at a rate proportional to the number of people that are infected at that time. If 1,000 people are infected when the epidemic is first discovered, and 1,200 are infected 7 days later, how many people are infected 12 days after the epidemic is first discovered?

$\frac{dy}{dx} = ky$

$C = 1000$

(A) 343
(B) 1,343
(C) 1,367
(D) 1,400
(E) 2,057

$y = C \cdot e^{kx}$

$y = 1000e^{kx}$

$(7, 1200)$

$1200 = 1000e^{k(7)}$

$1.2 = e^{7k}$

$\ln 1.2 = 7k$

$\frac{1}{7}\ln 1.2 = k$

Find $y(12)$

$y = 1000e^{\frac{1}{7}\ln(1.2) \cdot x}$

$y = 1000\, e^{\ln(1.2)^{\frac{x}{7}}}$

$y = 1000 \cdot 1.2^{\frac{x}{7}}$

$y(12) = 1000(1.2)^{12/7}$

$y(12) \approx 1366.908$

6. The weight of a population of yeast is given by a differentiable function y, where $y(t)$ is measured in grams and t is measured in days. The weight of the yeast population increases according to the equation $\frac{dy}{dt} = ky$, where k is a constant. At time $t = 0$, the weight of the yeast population is 120 grams and is increasing at the rate of 24 grams per day. Which of the following is an expression for $y(t)$?

(A) $120e^{24t}$
(B) $120e^{t/5}$
(C) $e^{t/5} + 119$
(D) $24t + 120$

$y = Ce^{kx}$

$C = 120$

$$\boxed{\begin{aligned} \frac{dy}{dx} &= k \cdot y \\ 24 &= k \cdot 120 \\ \frac{1}{5} &= k \end{aligned}}$$

Answer
$y = 120e^{\frac{1}{5}x}$

7. The population P of a city grows according to the differential equation $\frac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k ?

(A) 0.058
(B) 0.061
(C) 0.167
(D) 0.693
(E) 8.318

$C=1$
$P=2$
$t=12$

$P=Ce^{kt}$
$2=1\cdot e^{k(12)}$
$\ln 2 = 12k$
$\frac{1}{12}\ln 2 = k$
$.0578 \approx k$

8. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ ✗
(B) $2e^{kt}$
(C) $e^{kt}+3$ ✗
(D) kty+5 ✗
(E) $\frac{1}{2}ky^2 + \frac{1}{2}$ ✗

$y = Ce^{kx}$

Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is dy/dt=ky, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

C

9. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

$y = Ce^{kt}$

1

$y = 1{,}000{,}000e^{kt}$

when $t = 6$, $y = 500{,}000$

$500{,}000 = 1{,}000{,}000e^{k(6)}$

$\frac{1}{2} = e^{6k}$

$\ln\frac{1}{2} = 6k$

$\frac{1}{6}\ln\left(\frac{1}{2}\right) = k$

$\ln\left(\frac{1}{2}\right)^{1/6} = k$

2

$\frac{dy}{dt} = ky$

when $y = 600{,}000$ . . .

$\frac{dy}{dt} = \frac{1}{6}\ln\left(\frac{1}{2}\right) \cdot 600{,}000$

$\frac{dy}{dt} = 100{,}000 \cdot \ln\left(\frac{1}{2}\right)$

or

$\frac{dy}{dt} = -100{,}000\ln(2)$