4.2 – Straight-Line Motion: Connecting Position, Velocity and Acceleration Projectiles

A particle is moving along a horizontal line with position function as given. Perform an analysis of the particle's direction, acceleration, motion (speeding up or slowing down), and position by completing the given number lines.

1.) $s(t) = -t^3 + 9t^2 - 24t + 1$

$v(t) = -3t^2 + 18t - 24$

$v(t) = 0 \quad 0 = -3(t^2 - 6t + 8)$

$0 = -3(t-4)(t-2)$

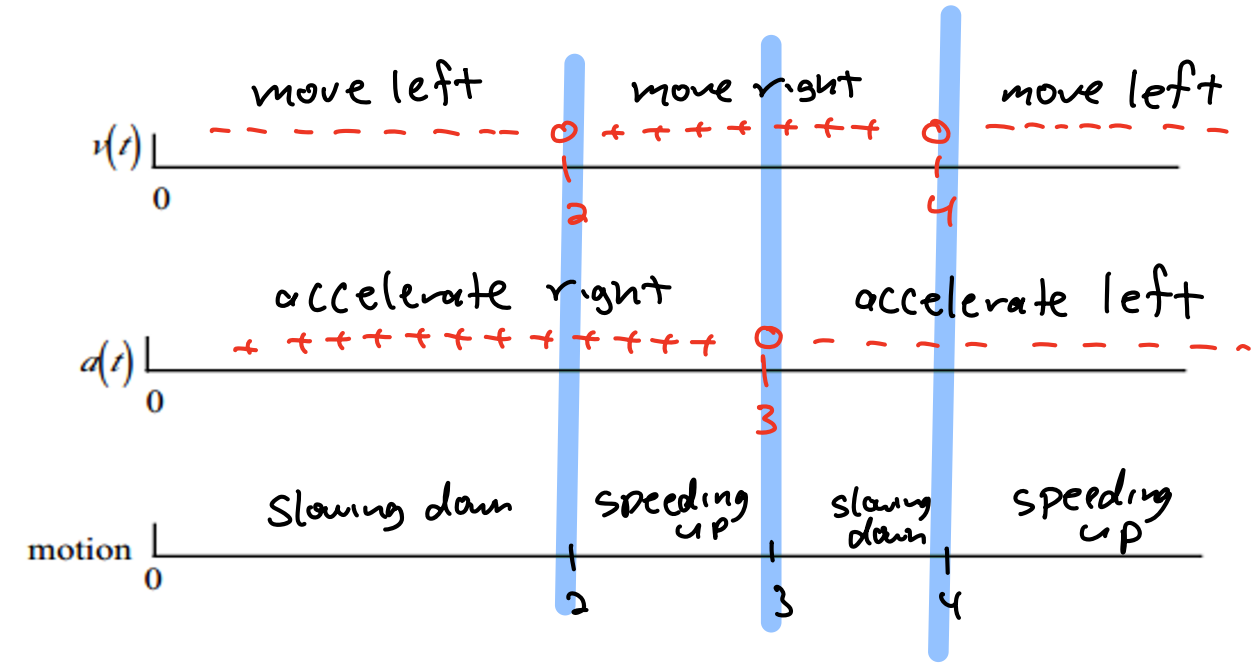
$t = 4, \; t = 2$

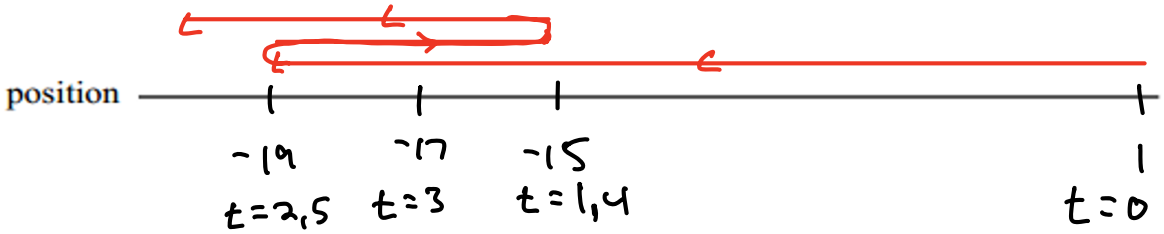
$a(t) = -6t + 18$

$a(t) = 0 \quad 0 = -6(t - 3)$

$t = 3$

t	s(t)
0	1
1	-15
2	-19
3	-17
4	-15
5	-19





2.) $s(t) = t + \frac{9}{t+1}$ $= t + 9(t+1)^{-1}$

$v = 1 - 9(t+1)^{-2}$

$v = 1 - \frac{9}{(t+1)^2}$

$v = \frac{(t+1)^2}{(t+1)^2} - \frac{9}{(t+1)^2}$

$v(t) = \frac{(t+1)^2 - 9}{(t+1)^2}$

$v(t) = \frac{t^2+2t+1-9}{(t+1)^2}$

$v(t) = \frac{t^2+2t-8}{(t+1)^2}$

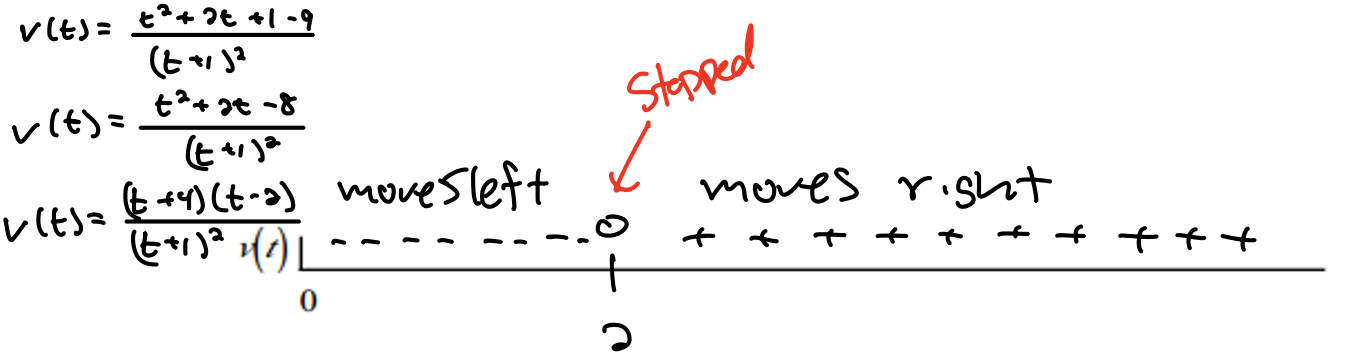
$v(t) = \frac{(t+4)(t-2)}{(t+1)^2}$

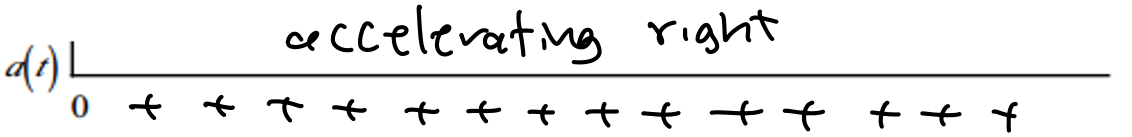
Set numerator = 0

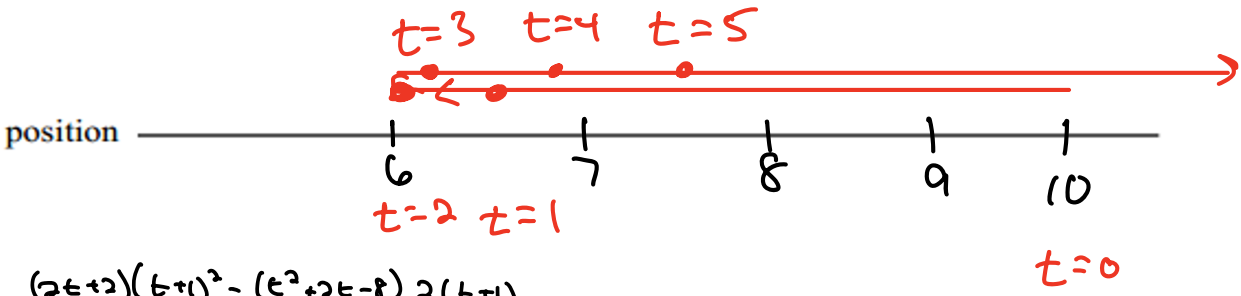
$v(t) = 0$ at $t \neq -4, t = 2$

$t \neq -1$ Not in domain

t	s(t)
0	10
1	6.5
2	6
3	6.25
4	6.8
5	7.5







$a(t) = \frac{(2t+2)(t+1)^2 - (t^2+2t-8)\,2(t+1)}{\left((t+1)^2\right)^2}$

GCF

$a(t) = \frac{(2t+2)\left[t^2+2t+1-t^2-2t+8\right]}{(t+1)^4}$

$a(t) = \frac{18(t+1)}{(t+1)^4}$

$a(t) = 0$ at $t \neq -1$ Numerator = 0

Not in Domain

3.) A dynamite blast propels a heavy rock straight up with a launch velocity of 160 ft/sec (about 109 mph). The rock reaches a height $s(t) = 160t - 16t^2$ feet after t seconds.

a.) How high does the rock go?

FIND MAXimum ∴ find when $v(t)=0$

$v(t) = 160 - 32t$

$v(t)=0$: $0 = 160(5-t)$

∴ $v(t)=0$ @ $t=5$

$s(5) = 400$ ft

The maximum height of the rock is 400 feet.

b.) What is the velocity of the rock when it is 256 ft above the ground

$256 = 160t - 16t^2$

$16t^2 - 160t + 256 = 0$

$16(t^2 - 10t + 16) = 0$

$16(t-8)(t-2) = 0$

$s(t) = 256$ @ $t = 2, t = 8$

i.) on the way up? $t=2$

$v(2) = 160 - 32(2)$

$v(2) = 96$ ft/sec

ii.) on the way down? $t=8$

$v(8) = 160 - 32(8)$

$v(8) = -96$ ft/sec

c.) What is the acceleration of the rock at any time t during its flight (after the blast)?

$a(t) = -32$ ft/sec^2

acceleration due to Earth's gravity

d.) When does the rock hit the ground?

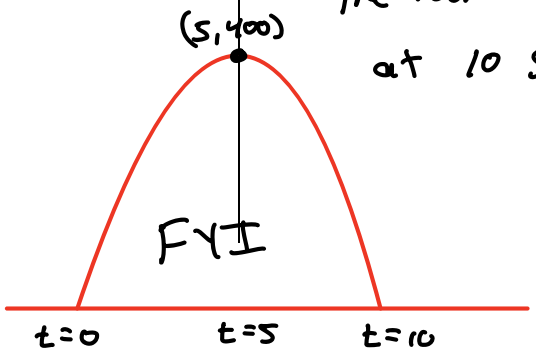
Rock is on ground ∴ $s(t) = 0$

$0 = 160t - 16t^2$

$0 = 16t(10 - t)$

$s(t) = 0$ @ $t = 0, t = 10$

The rock hits the ground at 10 seconds.



4.) A projectile is fired vertically upward with an initial velocity of 96 ft/sec from a tower 256 ft high. Therefore, $s(t) = -16t^2 + 96t + 256$.

a.) How long will it take for the projectile to reach its maximum height?

FIND MAximum ∴ find when $v(t) = 0$

$v(t) = -32t + 96$

$0 = -32(t - 3)$

$v(t) = 0$ @ $t = 3$

It will take 3 seconds to reach its maximum.

b.) What is its maximum height?

$s(3) = -16(3)^2 + 96(3) + 256$

$s(3) = 400$

The max height is 400 ft.

c.) How long will it take the projectile to reach its starting position on its way down?

$s(t) = 256$

$256 = -16t^2 + 96t + 256$

$0 = -16t(t - 6)$

$s(t) = 0$ @ $t = 0$, $t = 6$

First time, Second time

It will take 6 seconds to reach its starting height of 256 feet.

d.) What is the velocity of the projectile when it passes its starting point on the way down?

$v(6) = -32(6 - 3)$

$v(6) = -96$ ft/sec

velocity is -96 ft/sec when it passes its starting point.

e.) How long will it take for the projectile to hit the ground?

$s(t) = 0$

$0 = -16t^2 + 96t + 256$

$0 = -16(t^2 - 6t - 16)$

$0 = -16(t - 8)(t + 2)$

$s(t) = 0$ @ $t = -2$, $t = 8$

before it's fired?

It will take 8 seconds to hit the ground.

f.) What will the speed of the projectile be when it hits the ground?

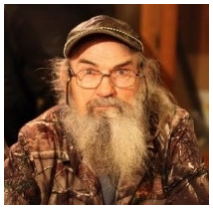
$v(8) = -32(8 - 3)$

$v(8) = -160$

$|-160|$ = speed

Speed is 160 ft/sec

5.) Uncle Si's four-wheeler runs out of gas as it goes up a hill. The vehicle rolls to a stop then starts rolling backwards. As it rolls, its displacement $d(t)$ in feet from the bottom of the hill at t seconds since the vehicle ran out of gas is given by $d(t) = 145 + 31t - t^2$.



a.) How far from the bottom of the hill was Uncle Si when he ran out of gas?

$t=0$ $d(0) = 145$ feet

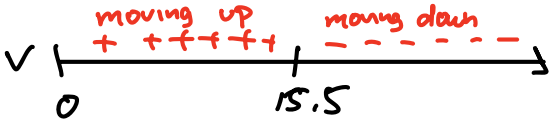
Uncle Si is 145 feet from bottom of hill when he ran out of grass

b.) When is his velocity positive? What does this mean in the context of the problem?

$v(t) = 31 - 2t$

$0 = 2(15.5 - t)$

$v(t) = 0$ @ $t = 15.5$



His velocity is positive from $t=0$ to $t = 15.5$ seconds. This is him coasting up hill after running out of gas

c.) How far was the four-wheeler from the bottom of the hill when it starts to roll backwards?

$d(15.5) = 145 + 31(15.5) - (15.5)^2$

$d(15.5) = 385.25$

He was 385.25 feet from bottom of hill.

d.) If Si keeps his foot off the brake, how long will it take for him to be at the bottom of the hill? → $d(t) = 0$ What will his speed be at that time?

$0 = 145 + 31t - t^2$

CALC

$t \approx 35.128$, $t \approx -4.128$

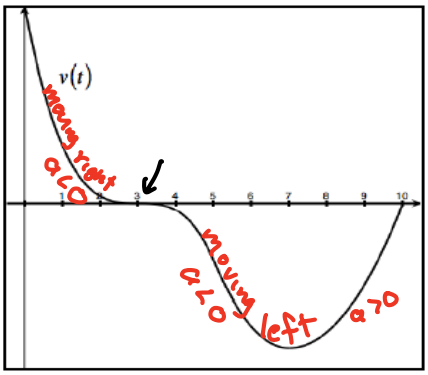
$v(35.128) \approx 31 - 2(31.128)$

$v(35.128) \approx -39.256$

Speed $= |-39.256|$

It will take him 35.128 seconds to coast to bottom of hill with a speed of 39.256 ft/sec

6.) The velocity $v(t)$ of a particle moving along the x-axis is shown in the figure to the right with t measured in seconds. Later in this course, you will learn ways to justify each response as well as finding how far the particle traveled.



a.) At what time t is the particle farthest to the right?

$t = 3$

b.) At what time intervals is the particle speeding up?

$v(t) < 0$ and $a(t) < 0$ on interval $(3, 7)$

$\therefore$ particle speeds up there.