4.1 – Interpreting the Meaning of the Derivative in Context

For each of the following scenarios, choose the appropriate choice that best answer the question.

1.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

N Continuous

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55 °F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected time t for the first 20 minutes are given in the table above.

minutes °F/min U

Using the data in the table, it is estimated that $W'(12) = 1.017$. Using correct units, interpret the meaning of your answer in the context of this problem.

T

(A) The temperature of the water at time t = 12 minutes is 1.017 $°F$.
(B) The water temperature is increasing at a rate of approximately 1.017 $°F$ per minute at time t = 12 minutes.
(C) The average temperature of the water at time t = 12 minutes is 1.017 $°F$.
(D) The change in the temperature of the water from time t = 0 to t = 12 minutes is 1.017 $°F$.

2.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

N

As a pot of tea cools, the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.
Use the data in the table to approximate the rate at which the temperature of the tea is changing at time t = 3.5.
Show the computations that lead to your answer and interpret its meaning in the context of the problem.
Which of the following would earn full credit on the AP Calculus Exam?

(A) $H'(3.5) = -\frac{8}{3}$. The temperature of the tea in the pot was decreasing by $-\frac{8}{3}$ °C per minute at time t = 3.5 minutes.

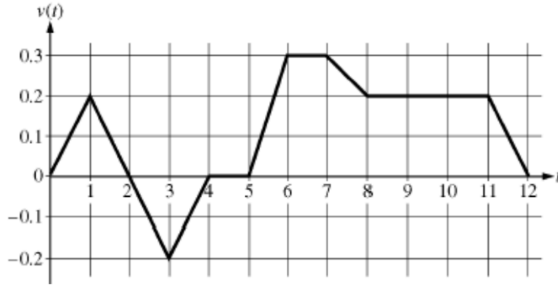
(B) $H'(3.5) = \frac{52-60}{5-2} = -\frac{8}{3}$. The temperature of the tea in the pot was decreasing by $-\frac{8}{3}$ °C per minute.

(C) $H'(3.5) = \frac{52-60}{5-2} = -\frac{8}{3}$. The temperature of the tea in the pot was decreasing by $\frac{8}{3}$ °C per minute at time t = 3.5 minutes.

N U T

(D) $H'(3.5) = \frac{52-60}{5-2} = -\frac{8}{3}$. The temperature of the tea in the pot was decreasing by $\frac{8}{3}$ °C per minute from time t = 2 to t = 5 minutes.

3.



N

Daronda rides her bicycle along a straight road from home to school, starting at home at time t = 0 minutes and arriving at school at time t = 12 minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute is modeled by the piecewise-linear function whose graph is shown above.

N

T

Find the acceleration of Daronda's bicycle at time t = 7.5 minutes. Indicate units of measure and interpret its meaning in the context of the problem.
Which of the following would earn full credit on the AP Calculus Exam?

$a(7.5) = mi/min^2$

U

y-axis is in tenths

(A) $a(7.5) = v'(7.5) = -1$ miles per minute per minute. Daronda's velocity was decreasing by 1 mile per minute per minute at time t = 7.5 minutes.

(B) $a(7.5) = v'(7.5) = \frac{0.2-0.3}{8-7} = -0.1$. Daronda's velocity was decreasing by 0.1 miles per minute at time t = 7.5 minutes.

(C) $a(7.5) = v'(7.5) = \frac{0.2-0.3}{8-7} = -0.1$. Daronda's velocity was decreasing by -0.1 miles per minute per minute at time t = 7.5 minutes.

(D) $a(7.5) = v'(7.5) = \frac{0.2-0.3}{8-7} = -0.1$. Daronda's velocity was decreasing by 0.1 miles per minute per minute at time t = 7.5 minutes.

N U

T

U

per

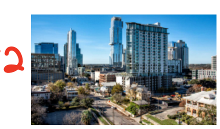
N

4. Let $P(t)$ represent a population of a city in millions of people and t represent the number of years since 2010. Which of the following statements correctly and thoroughly interprets the meaning of $P'(2) = \frac{1}{2}$?

T=2012

(A) The population in 2010 is growing by half a million people.
(B) The population in 2020 is growing by half a million people per year.
(C) The population in 2012 is growing by half a million people. rate? No
(D) The population in 2012 is growing by half a million people per year.

N T U



N U

5. Grass clippings are placed in a bin, where they decompose. For $0 \le t \le 30$, the amount of clippings remaining in the bin is modeled by $A(t) = 6.687(0.931)^t$, where $A(t)$ is measured in pounds and t is measured in days.

lbs/day U

a.) Find the average rate of change of $A(t)$ over the interval $0 \le t \le 30$. Indicate the units of measurement.

$$ARC = \frac{A(0) - A(30)}{0 - 30} \approx -.197 \text{ lbs/day}$$

N The grass clippings

U decompose at an average rate of 0.197 lbs per day

T over time period from t=0 to t=30

b.) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

$$A'(15) \approx -0.164 \text{ lbs/day}$$

N The grass clippings

U decompose at a rate of 0.164 lbs per day

T at t = 15 days.

N

6. A student proposes the function P, given by $P(\mathrm{t}) = 20 + 10\mathrm{te}^{-\frac{t}{3}}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units explain the meaning of your answer in the context of the problem.

$P' = {}^{\circ}C/\text{day}$

U

$t = 12$ days

T

$P'(12) \approx -0.549\ {}^{\circ}C/\text{day}$

N The temperature of water in pond

U is decreasing at rate of $0.549\ {}^{\circ}C/\text{day}$

T at $t = 12$ days.

N

7. For some painkillers, the size of the dose, D, given depends on the weight of the patient, W. Thus, $D = \mathrm{f(w)}$, where D is measured in milligrams and W is measured in pounds. For $f'(140) = 3$, explain the meaning of your answer in the context of the problem using appropriate units.

$f' = \frac{\text{mg}}{\text{lbs}}$ U

lbs

T

N The size of the dose of painkillers

U is increasing at a rate of 3 mg/(lbs of patient)

T when patient weighs 140 lbs.