Avon High School
AP Calculus AB

Name ______________________________

Period _____ Score ______ / 10

Skill Builder: Topic 1.6 – Determining Limits Using Algebraic Manipulation

Directions: Beginning in the first cell, find the answer. Search for that answer elsewhere in the document, mark that cell #2 and work the problem in that box. Process in this manner until you complete the circuit. Show all pertinent work. Calculators may not be used.
Note: This assignment contains two separate circuits.

Circuit 1

# 1 Answer: $\frac{-3}{8}$ $\lim_{x\to 5} 12$ 12 (The limit of a constant is a constant.)	# **5** Answer: 0 $\lim_{x\to 1}\frac{2x-2}{x-1}$ $\lim_{x\to 1}\frac{2x-2}{x-1}=\lim_{x\to 1}\frac{2(x-1)}{x-1}=\lim_{x\to 1} 2=2$
# **8** Answer: $\frac{3}{8}$ If $f(x)=\begin{cases} x-1, & x\le 3 \\ 2x-3, & x>3 \end{cases}$, find $\lim_{x\to 3} f(x)$. $\lim_{x\to 3^-}(x-1)=3-1=2$ $\lim_{x\to 3^+}(2(3)-3)=6-3=3$ $\lim_{x\to 3} f(x)$ does not exist	# **6** Answer: 2 $\lim_{s\to -1}\frac{s^2+6s+5}{s^2-3s-4}$ $\lim_{s\to -1}\frac{s^2+6s+5}{s^2-3s-4}=\lim_{s\to -1}\frac{(s+5)(s+1)}{(s-4)(s+1)}$ $=\lim_{s\to -1}\frac{s+5}{s-4}$ $=-\frac{4}{5}$
# **3** Answer: 8 $\lim_{x\to 5}(3x^2-4x-1)$ $\lim_{x\to 5}\left(3x^2-4x-1\right)=3(5)^2-4(5)-1$ $=75-20-1$ $=54$	# **11** Answer: $\frac{1}{4}$ $\lim_{t\to -2}\frac{\frac{1}{2}+\frac{1}{t}}{2+t}$ $=\lim_{x\to -2}\frac{\frac{t}{2t}+\frac{2}{2t}}{2+t}$ $=\lim_{x\to -2}\frac{t+2}{2t}\cdot\frac{1}{2+t}$ $=\lim_{x\to -2}\frac{1}{2t}=-\frac{1}{4}$

_10_ Answer: -1

$$\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$$

$$=\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}\cdot\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}$$

$$=\lim_{x\to 0}\frac{x+4-4}{x\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{x+4}+2}=\frac{1}{4}$$

_2_ Answer: 12

$$\lim_{x\to 2}4x$$

$$\lim_{x\to 2}4x=4(2)=8$$

_4_ Answer: 54

$$\lim_{x\to -2}\frac{x^2+4x+4}{x^2}$$

$$\lim_{x\to -2}\frac{x^2+4x+4}{x^2}=\frac{(-2)^2+4(-2)+4}{(-2)^2}$$

$$=\frac{4-8+4}{4}$$

$$=\frac{0}{4}=0$$

_9_ Answer: DNE

If $f(x)=\begin{cases}\cos x-\sin\pi, & x\le\pi\\ x-\pi-1, & x>\pi\end{cases}$, find $\lim_{x\to\pi}f(x)$.

$$\lim_{x\to\pi^-}(\cos\pi-\sin\pi)=-1$$

$$\lim_{x\to\pi^+}(\pi-\pi-1)=-1$$

$$\lim_{x\to\pi}f(x)=-1$$

_7_ Answer: $\frac{-4}{5}$

$$\lim_{x\to 2}\frac{x^3+x^2-4x-4}{x^4-16}$$

$$=\lim_{x\to 2}\frac{x^2(x+1)-4(x+1)}{x^4-16}$$

$$=\lim_{x\to 2}\frac{(x+1)(x^2-4)}{(x^2-4)(x^2+4)}$$

$$=\lim_{x\to 2}\frac{x+1}{x^2+4}$$

$$=\frac{3}{8}$$

_12_ Answer: $\frac{-1}{4}$

$$\lim_{x\to 1}\frac{5x^2-2x^3+2x-5}{4x^2+6x-10x^3}$$

$$\lim_{x\to 1}\frac{-2x^3+5x^2+2x-5}{-10x^3+4x^2+6x}$$

$$=\lim_{x\to 1}\frac{-x^2(2x-5)+1(2x-5)}{-2x(5x^2-2x-3)}$$

$$=\lim_{x\to 1}\frac{(2x-5)(1-x^2)}{-2x(5x+3)(x-1)}$$

$$=\lim_{x\to 1}\frac{-(2x-5)(x-1)(x+1)}{-2x(5x+3)(x-1)}$$

$$=\lim_{x\to 1}\frac{(2x-5)(x+1)}{2x(5x+3)}=\frac{(-3)(2)}{2(8)}=-\frac{3}{8}$$

Circuit 2

# 1 Answer: -1 $\lim_{x\to 0}\pi$ $=\pi$	# _**5**_ Answer: $\frac{4}{3}$ $\lim_{x\to 4}\frac{x^2-16}{x-4}$ $=\lim_{x\to 4}\frac{(x-4)(x+4)}{x-4}$ $=\lim_{x\to 4}(x+4)$ $=8$
# _**7**_ Answer: 12 $\lim_{x\to 2}\frac{x^2-4x+4}{x^2+x-6}$ $=\lim_{x\to 2}\frac{(x-2)(x-2)}{(x+3)(x-2)}$ $=\lim_{x\to 2}\frac{x-2}{x+3}$ $=\frac{0}{5}=0$	# _**4**_ Answer: 11 $\lim_{x\to 4}\frac{2x-4}{x-1}$ $=\frac{2(4)-4}{4-1}$ $=\frac{4}{3}$
# _**2**_ Answer: π $\lim_{x\to 1}\left(5x^3-7x^2+2^x-2\right)$ $=5(1)^3-7(1)^2+2^1-2$ $=-2$	# _**12**_ Answer: $2\sqrt{3}$ $\lim_{x\to 1}\left(\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right)$ $=\lim_{x\to 1}\frac{x^2-3x+2}{(x-1)(x^2-3x+2)}+\frac{x-1}{(x-1)(x^2-3x+2)}$ $=\lim_{x\to 1}\frac{x^2-2x+1}{(x-1)(x^2-3x+2)}$ $=\lim_{x\to 1}\frac{(x-1)(x-1)}{(x-1)(x-1)(x-2)}$ $=\lim_{x\to 1}\frac{1}{x-2}$ $=-1$

_11_ Answer: $\frac{1}{4}$

$$\lim_{h\to 0}\frac{h^2}{\sqrt{h^2+h+3}-\sqrt{h+3}}$$

$$=\lim_{h\to 0}\frac{h^2}{\sqrt{h^2+h+3}-\sqrt{h+3}}\cdot\frac{\sqrt{h^2+h+3}+\sqrt{h+3}}{\sqrt{h^2+h+3}+\sqrt{h+3}}$$

$$=\lim_{h\to 0}\frac{h^2\left(\sqrt{h^2+h+3}+\sqrt{h+3}\right)}{h^2+h+3-(h+3)}$$

$$=\lim_{h\to 0}\frac{\sqrt{h^2+h+3}+\sqrt{h+3}}{1}$$

$$=2\sqrt{3}$$

_10_ Answer: DNE

$$\lim_{x\to 0}\frac{\sqrt{x^2+x+4}-2}{x^2+x}$$

$$=\lim_{x\to 0}\frac{\sqrt{x^2+x+4}-2}{x^2+x}\cdot\frac{\sqrt{x^2+x+4}+2}{(x^2+x)\left(\sqrt{x^2+x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{x^2+x+4-4}{(x^2+x)\left(\sqrt{x^2+x+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{x^2+x+4}+2}$$

$$=\frac{1}{4}$$

_6_ Answer: 8

$$\lim_{t\to -2}\frac{t^3+8}{t+2}$$

$$=\lim_{x\to -2}\frac{(t+2)(t^2-2t+4)}{t+2}$$

$$=\lim_{x\to -2}(t^2-2t+4)$$

$$=(-2)^2-2(-2)+4$$

$$=4+4+4=12$$

_8_ Answer: 0

$$\lim_{x\to 3}\frac{2x^3-54}{x-3}$$

$$=\lim_{x\to 3}\frac{2(x-3)(x^2+3x+9)}{x-3}$$

$$=\lim_{x\to 3}\left[2(x^2+3x+9)\right]$$

$$=2(3^2+3(3)+9)$$

$$=54$$

_9_ Answer: 54

If $f(x)=\begin{cases}x^2+x, & x\le -1\\ -2^{-x}, & x>-1\end{cases}$, find $\lim_{x\to -1} f(x)$.

$$\lim_{x\to -1^-}\left((-1)^2+(-1)\right)=0$$

$$\lim_{x\to -1^+}(-2^{-(-1)})=-2$$

$$\therefore \lim_{x\to -1} f(x) \text{ does not exist}$$

_3__ Answer: −2

$$\lim_{y\to -1}(3y^4-6y^3-2y)$$

$$=3(-1)^4-6(-1)^3-2(-1)$$

$$=3+6+2$$

$$=11$$